Evidence for Internal Structures of Spiral Turbulence

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Abstract

We report the first three-dimensional direct numerical simulation of spiral turbulence in a Taylor-Couette geometry. Conditionally averaged flow statistics show that the spiral turbulence possesses fairly complex internal structures. It is shown that a significant velocity gradient persists along the azimuthal direction, and that the cores of turbulent and laminar spirals mark interfaces that separate the entire flow domain into regions of fluids that tend to move in axially-opposite directions.

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Co-existence of turbulent and laminar domains in space and time is one of the most fascinating phenomena in fluid dynamics. Spatiotemporal intermittency and pattern formation in such flows have been observed in a variety of systems as well as in theoretical studies of partial differential equations (see [1] and the references therein). Particularly intriguing is the spiral turbulence regime (or the barber-pole pattern commented on by Feynman [2]) in the Taylor-Couette setting where intertwined helical turbulent and laminar stripes propagate between counter-rotating concentric cylinders [3–9]. On the largest scale spiral turbulence has been observed to relate to a finite-wavelength modulation of turbulent intensities [9]. It is also observed that aspects of the spiral pattern can be qualitatively described by model equations [6, 9, 10]. In other types of flow systems related patterns are the stationary laminar-turbulent patterns in plane Couette flow [9, 11] and the torsional flow between a stationary and a rotating disk [12].

In this letter we report the first three-dimensional (3D) direct numerical simulation of spiral turbulence. By employing a conditional averaging technique, we have obtained flow statistics of a single turbulent/laminar spiral that is “frozen” in space. It is revealed that the helical patterns of turbulent and laminar spirals possess fairly complex internal structures. Large-scale spatial variations have been observed for the flow internal to the spirals.

More specifically, we consider the incompressible flow between two infinitely long concentric cylinders. The cylinder axis is aligned with the z-axis of the coordinate system. The geometry is characterized by the radius ratio, \( \eta = R_i/R_o \), where \( R_i \) and \( R_o \) are respectively the radii of the inner and outer cylinders, and the aspect ratio, \( \Gamma = L_z/d \), where \( L_z \) is the axial dimension of the domain and \( d \) is the gap width, \( d = R_o - R_i \). The inner cylinder rotates counter-clockwise (when viewed toward \(-z\)-direction) at an angular velocity \( \Omega_i \) (\( \Omega_i > 0 \)), while the outer cylinder rotates clockwise at an angular velocity \( \Omega_o \) (\( \Omega_o < 0 \)). The length variables are normalized by \( R_i \), and the velocity by a natural velocity \( U_d \) such that the normalized outer cylinder velocity is \( \Omega_o R_o / U_d = -1.08 \). The inner- and outer-cylinder Reynolds numbers are defined by \( Re_i = \Omega_i R_i d/\nu \) and \( Re_o = \Omega_o R_o d/\nu \), where \( \nu \) is the fluid kinematic viscosity.

We numerically solve the 3D Navier-Stokes equations employing a Fourier spectral expansion of flow variables along the axial direction, assuming a periodicity of flow at \( z = 0 \) and \( z = L_z \), and a high-order spectral-element discretization of the annular domain [13]; The effectiveness of spectral element-based approach has been evidenced in studies of several
FIG. 1: Evolution of turbulent-laminar patterns in space \((z)\) and time \((t)\). Plotted are contours of the azimuthal velocity along a line parallel to the cylinder axis and fixed in the mid-gap. Time evolves horizontally with changes in \(Re_i\) indicated at the top \((Re_o\) is fixed at \(-1375\)). With increasing \(Re_i\) states seen from intermittent turbulent bursts \((Re_i = 530)\), through spiral turbulence, ending in complete turbulence \((Re_i = 900)\). Dark and blank regions respectively represent turbulent and laminar flows.

types of flows [14]. The time integration is based on a stiffly stable scheme [15]; Dirichlet boundary conditions are applied on the inner- and outer-cylinder walls to reflect their respective rotation velocities.

We have considered a radius ratio \(\eta = 0.89\), comparable to those of several experiments [3, 5, 6]. We fix \(Re_o\) at \(-1375\), while \(Re_i\) is varied between 530 and 900. In light of the computational cost involved, we aim to simulate only one complete turbulent spiral. For this purpose we have considered several values of the axial dimension ranging from \(\Gamma \approx 6\) to 25. At a low \(\Gamma\) only turbulent patches can be observed; At \(\Gamma \approx 12\) and higher complete turbulent spirals are observed. Results reported herein are for an aspect ratio \(\Gamma = 25.1\). Therefore, the setting for the present simulations resembles more the experiments of [3, 5, 6], while the experiment of [9] is with very large aspect ratios \((\sim 400)\). The number of Fourier planes in the axial direction is varied between 256 and 512. In the annular domain 640 quadrilateral spectral elements are employed; To ensure convergence of computation results
we have systematically varied the element order, from order 6 to 10, for the resolution test. By comparing the torque on cylinder walls and the mean and root-mean-square (r.m.s.) fluctuation flow profiles [13] at different resolutions, we have confirmed the convergence of the simulation results. Our application code has been extensively validated for Taylor-Couette turbulence by comparisons of the computed torque and flow statistics with those determined from experiments [16]; see [13] for details on the comparisons and validations.

We start by exploring the Reynolds number dependence of the patterns. Figure 1 shows a composite plot of a long simulation spanning the range \(530 \leq Re_i \leq 900\), with \(Re_i\) increased in discrete steps, while \(Re_o\) is fixed at \(-1375\). Shown are the stable patterns at each Reynolds number (Transients at the change of \(Re_i\) are not shown). Plotted in the figure are spatial-temporal (z-t) contours of the azimuthal velocity along a line in the axial direction that is fixed in the mid-gap.

Distinct flow patterns can be identified with the increase of \(Re_i\). At \(Re_i = 530\), patches of turbulent regions (turbulent bursts) are observed to emerge from the laminar flow background, persist for a period of time, and then disappear into the flow; The differences between the bursts observed here and those discussed in [17] are that here they are localized in both space and time while those in [17] fill the entire domain and represent a temporal oscillation of laminar and turbulent states. The advent of turbulent bursts is quasi-periodic in time (Figure 1); At times turbulent patches form a nearly complete spiral, and then abruptly vanish into the laminar flow environment. At \(Re_i = 560 \sim 700\), regular alternating turbulent-laminar spiral patterns are observed, characterized by regularly-spaced inclined stripes in the spatial-temporal diagram (Figure 1). As \(Re_i\) further increases to 750 and 800, although the spiral pattern can still be vaguely recognized, the flow is permeated with increasing turbulent fluctuations. At \(Re_i = 900\) the entire flow becomes fully turbulent, and no apparent large-scale patterns can be discerned.

We next concentrate on \(Re_i\) values at which well-defined turbulent-laminar patterns can be observed. Figure 2(a) shows a typical turbulent-laminar spiral pattern obtained from the simulations. We have plotted contours of the instantaneous azimuthal velocity in a grid surface (essentially cylindrical) located near the mid-gap; One can observe that the turbulent and laminar regions, characterized respectively by rapid/rough and slow/smooth variations in the azimuthal velocity, form a left-handed spiral. This plot is reminiscent of the flow photographs of turbulent spirals in previous experiments (see e.g. [5]). The spiral
FIG. 2: A left-handed turbulent spiral ($Re_i=611$, $Re_o=-1375$): (a) Contours of instantaneous azimuthal velocity in a cylindrical grid surface near the mid-gap. (b) Iso-surface of conditionally-averaged r.m.s. velocity magnitude $u'/U_d = 0.14$. The inner cylinder is also shown in (b).

pattern rotates clockwise when viewed toward the $-z$-direction, in the same direction as the outer cylinder. In fact, the pattern rotates in the same direction as the outer cylinder for all the Reynolds numbers considered here, a point consistent with and also noted by previous experiments [4–6, 8]. Right-handed spirals have also been observed in the simulations (e.g. at $Re_i = 700$).

To explore the statistical characteristics of turbulent spirals, we employ a conditional averaging technique. Specifically, for a given $Re_i$ the rotation period (or angular frequency) of the spiral pattern is first determined based on the fast Fourier transform (FFT) of a long-time history of the velocity data. We then pick a time instant $t_0$, and consider the flow field at $t_0$ as the base flow for averaging. We consider a moving coordinate system which coincides with the laboratory coordinate system at $t_0$ and rotates around the $z$-axis at the same angular frequency as the spiral pattern. At any time $t$ ($t > t_0$), we rotate the flow field back to the base configuration (i.e., that at $t_0$) based on $(t - t_0)$ and the angular frequency of the spiral pattern. The rotated field is then accumulated to the base flow for averaging. With this technique the spiral pattern has been essentially frozen in space, and its key statistical features can be exposed. Figure 2(b) shows in 3D space an iso-surface of the conditional r.m.s. velocity magnitude $u'/U_d = 0.14$ at $Re_i = 611$, where $u' = \sqrt{u'^2 + u'^2_\theta + u'^2_z} (u'_r, u'_\theta)$.
FIG. 3: Conditional mean, r.m.s., and instantaneous velocities in the mid-gap ($Re_i=611$, $Re_o = -1375$): (a), azimuthal velocity; (b), axial velocity.

and $u'_r$, $u'_\phi$, and $u'_z$ denoting respectively the conditional r.m.s. velocity in radial, azimuthal and axial directions). The iso-surface approximately reflects the shape of the 3D interface (in the mean sense) between the turbulent and laminar regions. The domain enclosed by the iso-surface marks the region of turbulence, a wide flat helical band wrapping around the inner cylinder. The leading edge of the turbulent spiral (i.e. the lower edge in Figure 2b, since it rotates clockwise) is radially nearer to the outer cylinder, while the trailing edge is radially closer to the inner cylinder, a topological feature that will become clearer later.

Another type of conditional averaging has also been employed to complement the whole-field averaging described above. It is applied to the spatial-temporal flow data (see Figure 1 for spatial-temporal diagram). For a given $Re_i$, the average inclination angle of the stripe pattern in the spatial-temporal diagram is first determined. We then pick an axial location $z_0$ as the base location, and consider the velocity history at $z_0$ as the base velocity signal for averaging. The velocity history at any axial location $z$ is then shifted in time based on $(z-z_0)$ and the stripe average inclination angle in spatial-temporal diagram, and accumulated to the base signal for averaging. Then, the conditionally-averaged velocity history (at $z_0$) is shifted in time by multiples of the rotation period of the spiral pattern, and averaged over different rotation periods. We will refer to this procedure as the spatiotemporal conditional averaging hereafter.
The conditionally averaged flow statistics show that the spiral turbulence has fairly complex internal structures. Firstly, a persistent azimuthal velocity gradient, \( \frac{\partial \langle u_\theta \rangle}{\partial \theta} \) (where \( \langle u_\theta \rangle \) denotes conditionally-averaged azimuthal velocity, and \( r \) and \( \theta \) are respectively the radial and azimuthal coordinates), exists in the turbulent and the laminar spiral regions. Figure 3(a) shows several periods in time of the spatiotemporally averaged conditional mean and conditional r.m.s. azimuthal velocity in the mid-gap at \( Re_i = 611 \) (a left-handed spiral), together with a time history signal of the instantaneous azimuthal velocity. Alternating turbulent and laminar regions can be clearly distinguished. Turbulent regions are characterized by high-frequency fluctuations in the instantaneous velocity and high values of conditional r.m.s. velocity, while the laminar regions are generally void of rapid instantaneous fluctuations and with low (nearly zero) r.m.s. velocity. Most striking is the systemic variations in the conditionally averaged mean velocity curve, which shows the presence of a significant azimuthal velocity gradient across a turbulent spiral along the azimuthal direction; From the leading edge to the trailing edge (see Figure 2b) the mean azimuthal velocity decreases notably, indicating a larger azimuthal velocity magnitude at the trailing edge. In the laminar spiral region the reverse is true. We have also examined the statistics data at other radial locations for \( Re_i = 611 \), and for other Reynolds numbers with right-handed spirals (e.g. \( Re_i = 700 \)), which lead to the same conclusions.

Secondly, the turbulent and laminar spirals are markers that separate regions of fluids moving in axially opposite directions. Figure 3(b) shows several periods in time of the spatiotemporally averaged conditional mean and conditional r.m.s. axial velocity in the mid-gap for a left-handed spiral (\( Re_i = 611 \)), together with a time history signal of the instantaneous axial velocity. From the leading edge to the trailing edge of a turbulent spiral, the conditional mean axial velocity changes sign, showing that axially the flow tends to be away from the middle of the turbulent region. In the laminar spiral region, on the other hand, the flow tends to be toward the middle of the region. This point is more clearly illustrated by the conditionally averaged whole-field data. In Figure 4(a) we plot contours of the conditional mean axial velocity in a radial-axial plane. Comparing this plot with Figure 4(b), which shows contours of the conditional r.m.s. velocity magnitude \( u'/U_d \) in the same radial-axial plane, we observe that the core of the turbulent spiral marks an inclined interface (with respect to a plane normal to the axial direction) separating two regions of fluids that tend to move away from the interface. Similarly, the middle of the
FIG. 4: Contours of conditional mean axial velocity $u'_z/U_d$ (a), conditional rms velocity magnitude $u'/U_d$ (b) in a radial-axial plane, and of the conditional r.m.s. velocity magnitude $u'/U_d$ in a horizontal plane at the mid-height of cylinder (c) ($Re_i=611$, $Re_o=-1375$).

laminar region marks another interface at which the upward- and downward-going fluids move toward each other. By examining the data at different Reynolds numbers, we have confirmed that this conclusion also applies to right-handed spiral patterns. However, for a left-handed turbulent spiral the leading (respectively, trailing) edge is associated with a negative (resp. positive) mean axial velocity, while for a right-handed spiral the leading (resp. trailing) edge is associated with a positive (resp. negative) mean axial velocity. Note that the pattern rotates in the same direction as the outer cylinder for both types of spirals.

In addition, the distribution of turbulent intensities in spiral turbulence exhibits a generally different characteristic than that in fully-developed turbulence. This is demonstrated by Figure 4(b), and Figure 4(c) in which we plot contours of the conditional r.m.s. velocity magnitude $u'/U_d$ in a horizontal $x-y$ plane at mid-height of the cylinder. In spiral turbulence the most energetic intensity is at the core of turbulent spiral, i.e. toward the middle of the gap. In contrast, in fully-developed Taylor-Couette turbulence at high Reynolds numbers
the strongest turbulent intensity tends to be found in regions near both walls rather than in the mid-gap [13]. Figures 4(b) and (c) also clearly illustrate our previous point that the leading edge of the turbulent spiral (bottom side in Figure 4b, bottom-left tip in Figure 4c) has a proximity to the outer wall while the trailing edge (top side in Figure 4b, right-top tip in Figure 4c) has a proximity to the inner wall, a point also noted by [4].

To summarize, 3D direct numerical simulations have provided new insights into the spiral turbulence. The simulation has shown, for the first time, that turbulent and laminar spirals possess fairly complex internal structures. A new conceptual model for spiral turbulence emerges from the simulation data. In this model, the leading and the trailing edges of the turbulent spiral are respectively associated with low and high azimuthal velocity magnitudes (in the mean sense), and a significant velocity gradient persists along the azimuthal direction in the turbulent and the laminar helical regions. The cores of the turbulent and laminar spirals correspond to two interfaces, inclined with respect to the plane normal to the axial direction, which separates regions of fluids moving in opposite directions axially; The fluids tend to move away from the interface of the turbulent spiral and toward that of the laminar one. In the present setting we have simulated only one period of the helical pattern in the axial direction. With large aspect ratios such as that of the experiment [9], in which multiple periods of the spiral pattern or multiple spirals are realized, the cores of adjacent turbulent and laminar spirals would separate the entire flow domain into different regions. Within each individual region the fluid tends to move axially away from the turbulent core and toward the laminar core.

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