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EXPLOITING NONLINEARITY TO PROVIDE BROADBAND ENERGY HARVESTING

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ABSTRACT

Energy harvesters are a promising technology for capturing useful energy from the environment or a machine's operation. In this paper we highlight ideas that might lead to energy harvesters that more efficiently harvest a portion of the considerable vibrational energy that is present for human-made devices and environments such as automobiles, trains, aircraft, watercraft, machinery, and buildings. Specifically, we consider how to exploit ideas based on properties of nonlinear oscillators with negative linear stiffness driven by periodic and stochastic inputs to design energy harvesters having large amplitude response over a broad range of ambient vibration frequencies. Such harvesters could improve upon proposed harvesters of vibrational energy based on linear mechanical principles, which only give appreciable response if the dominant ambient vibration frequency is close to the resonance frequency of the harvester.

INTRODUCTION

Typical vibrational energy harvesters are composed of a mass-spring system with a transducer [1,2], where vibrations in the surrounding environment act as inputs and cause the spring-mass system to oscillate. The oscillations of the device are converted into electric energy by electrostatic, piezoelectric, or electromagnetic transduction [1,2]. Proposed harvesters of vibrational energy are typically based on linear mechanical principles, e.g., [1–3]. Such devices give appreciable response amplitude only if the dominant ambient vibration frequency is close to the resonance frequency of the harvester. In order to achieve maximum conversion efficiency, the dominant ambient vibration frequency must therefore be known prior to the design process. For a broadband or time-varying ambient vibration spectrum, only a small fraction of the available ambient vibration energy can be extracted by such devices.

Improving the bandwidth of vibrational energy harvesters is crucial for increasing their efficiency and functionality. There have been attempts to overcome the bandwidth limitations while staying within the linear mechanical system framework, e.g., [4–7], and recently attempts have been made to exploit nonlinearities for energy harvesting, e.g., [8–13]. In this paper we describe the promising approach of exploiting nonlinear effects for oscillators with negative stiffness to increase the range of vibration frequencies which give large amplitude response. Note that others have recently investigated the use of negative stiffness oscillators for energy harvesting applications [9,10,14].

We envision implementation using microelectromechanical oscillators, but we use a very general class of oscillators in a vibrating environment for illustration:

$$m\ddot{x} - F_d(x,\dot{x}) - F_r(x) = -m\ddot{f}. \tag{1}$$

Here m is the mass of the oscillator, F_d is the damping force for

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the device (due, for example, to friction, air resistance, or transduction of power), F_r is the restoring force for the oscillator (due, for example, to spring-like mechanical forces and electrostatic forces), and f is the displacement of the base of the oscillator due to vibration [1]. Without loss of generality, we take $x = 0$ to correspond to the oscillator being at an equilibrium position (which might be unstable). We define $\alpha = -\frac{\partial F_r}{\partial x}\Big|_{x=0}$ to be the linear stiffness of the device. The key idea is to design or tune a nonlinear oscillator so that its linear stiffness α is negative, as this will allow the device to respond with large amplitude over a very broad range of frequencies, as illustrated below.

PERIODIC FORCING

A periodically forced oscillator obeying (1) with $\alpha < 0$ can undergo various types of large-amplitude oscillations, including chaotic oscillations (which can be transient or attracting), large-amplitude periodic oscillations, and large-amplitude quasiperiodic oscillations. The behavior depends on the design of the device, the frequency and amplitude of the forcing, and the damping, which will be influenced by the mechanism used to transduce mechanical energy into usable electrical energy. For example, consider an oscillator with

$$F_d(x,\dot{x}) = -\delta\dot{x}, \qquad F_r(x) = x - x^3, \qquad f = \frac{\gamma}{\omega^2}\cos(\omega t), \quad (2)$$

corresponding to linear positive damping ($\delta > 0$), a nonlinear restoring force with $\alpha = -1$, and periodic vibration of the base of the oscillator of amplitude γ and frequency ω. For simplicity, we suppose that $m = 1$. With these forces, the system corresponds to the forced Duffing oscillator; see, for example, [10,15]. Figure 1 illustrates that large amplitude response, defined here as oscillations that have some values of $x > 1$ and $x < -1$, occurs over a very broad range of driving frequencies. We observe that large response extends down to very low frequencies.

It is important to mention that Figure 1 only displays single attractors at each value for ω. There might be co-existing large amplitude attractors that the sweeping procedure used to generate Figure 1 missed. Furthermore, there could be transient behavior which is also of large amplitude. It is well known, for example, that transient chaotic behavior can occur for parameter values for which no chaotic attractor exists [15]. When such a chaotic transient corresponds to a large amplitude oscillation, it can also lead to efficient energy harvesting.

An interesting viewpoint for understanding the large response over a broad frequency range is the following. An oscillator tuned to have a negative linear stiffness might be in a regime in which it can undergo chaotic oscillations, which could be transient or attracting. Now, it is known that embedded within a chaotic set are an infinite number of unstable periodic orbits, each of which generically has a different frequency [15]. Indeed, chaos can be viewed as the system "bouncing around" amongst

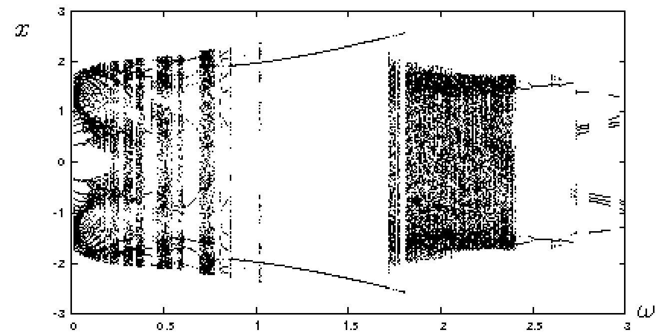

Figure 1. Bifurcation diagram showing the response of the forced Duffing oscillator with $\delta = 0.1, \gamma = 1$. For each value of ω, we integrate to get rid of transients, then plot the instantaneous value of x whenever $\dot{x} = 0$. We see that the oscillator undergoes large amplitude oscillations for a wide range of forcing frequencies ω.

these unstable periodic orbits; this is why the power spectrum for a chaotic signal is broadband [16]. The response of oscillators in the chaotic regime might be related to resonances between the drive frequency and the unstable periodic orbits embedded in the chaotic set.

STOCHASTIC FORCING

A stochastically forced oscillator with $\alpha < 0$ can also undergo large-amplitude oscillations between co-existing quasi-stable equilibria. To understand this, we first consider the system in the absence of damping and forcing, that is when $F_d(x,\dot{x}) = 0$ and $\ddot{f} = 0$. In this limit, the dynamics take the form

$$\ddot{x} = -\frac{dV}{dx}, \qquad V(x) = \frac{1}{4}x^4 - \frac{1}{2}x^2. \quad (3)$$

This is a double well potential with minima at $x = -1$ and $x = 1$, and a local maximum at $x = 0$, the former corresponding to stable equilibria and the latter to an unstable equilibrium. Now consider the stochastically forced oscillator $\ddot{x} + \delta\dot{x} - x + x^3 = \eta(t)$, where $\eta(t)$ represents a Gaussian white noise random process with the properties $\langle\eta(t)\rangle = 0$, $\langle\eta(t)\eta(t')\rangle = 2D\delta(t-t')$. Figure 2 shows an example time series for the displacement x when $\delta = 0.1$ and $D = 0.01$. Here the stochastic forcing causes noise-induced transitions between neighborhoods of the minima, giving large amplitude oscillations [11]. Such transitions might be enhanced by tuning and driving the system to exploit stochastic resonance [9].

IMPLEMENTATION USING MEMS DEVICES

Achieving desired values of α can be accomplished by electrostatically tuning MEMS oscillators through the tuning scheme described in [17,18]. For example, a shuttle mass oscillator with

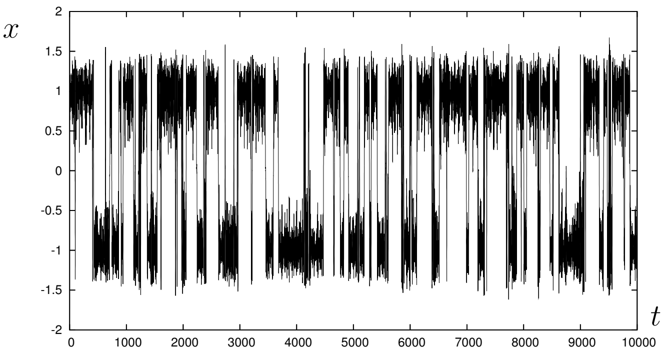

Figure 2. Noise-induced transitions between potential wells with minima at $x = \pm 1$, leading to large amplitude oscillations.

fixed-fixed springs will have a restoring force $F_r(x) = k_1 x + k_3 x^3$, where k_1 is the linear mechanical stiffness and k_3 is the nonlinear stiffness that arises due to the stretching of each spring's neutral axis, which results from the boundary conditions. The effective stiffness can be tuned by using a set of noninterdigitated combfingers. One side of the combfingers is attached to a static electrode and the other is attached to the shuttle mass. By applying a DC voltage (V_{DC}) across the combfingers, an electrostatic force $F_{es}(x) = \left(r_1 x + r_3 x^3\right) V_{DC}^2$ is produced, where r_1 is the linear electrostatic stiffness and r_3 is the nonlinear electrostatic stiffness. By having the combfingers misaligned with respect to each other, and designing the combfinger gap, spacing, and width appropriately the linear electrostatic stiffness can be negative [18, 19]. As a result, the effective linear stiffness $\alpha = \left(k_1 + r_1 V_{DC}^2\right)$ can be tuned by adjusting V_{DC} and can be small positive, zero $\left(\text{for } V_{DC} = \sqrt{-k_1/r_1}\right)$, or negative.

The vibrational energy of such an oscillator could be transduced into electrical energy using standard methods such as electrostatic (capacitive), piezoelectric, or electromagnetic (inductive) [1, 2]. We note that energy is required to tune the linear stiffness α; however, if the amount of harvested energy exceeds this energy, then the harvester gives a net positive contribution.

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