

# ME225BC: Introduction to Low-Dimensional Modeling Winter 2004

## Important Information

Lectures: Monday, Wednesday, and Friday, 1:00-1:50, starting Jan. 5  
Bldg 387, Room 101

**No lecture:** Friday, Jan 16 (make-up to be scheduled)  
Monday, Jan 19 (MLK Jr Birthday)  
Friday, Feb 13 (make-up to be scheduled)  
Monday, Feb 16 (President's Day)

Course Webpage: <http://www.engineering.ucsb.edu/~moehlis/ME225>

Questions? Contact (email preferred):

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## Course Description

Many biological and physical systems of interest have fine details which may not be necessary for a basic understanding of the system's behavior. Through appropriate simplifications, low-dimensional mathematical models may be constructed which capture qualitative, and perhaps also quantitative, aspects of the system's dynamics. This course will cover the development and analysis of such models for problems arising in neuroscience and, as time permits, fluid dynamics. More broadly, it will teach the student a variety of mathematical modeling and analysis techniques which can be applied to problems from engineering, physics, chemistry, and biology. Specific topics to be covered include:

### Neuroscience Modeling:

- basics of neuroscience
- conductance-based neuron models, such as the Hodgkin-Huxley equations
- simple reductions of conductance-based neuron models
- nullecline analysis of neuron models
- integrate-and-fire neuron models
- phase response curves and isochrons for periodically firing neurons
- dynamics of coupled neurons
- reduction of coupled neuron systems to phase models
- analysis of phase-locked solutions for coupled neuron systems
- analysis of bursting neuron models

## Fluid Dynamics Modeling, as time permits:

- basics of fluid dynamics
- derivation of low-dimensional models using Galerkin projection
- analysis of the Lorenz equations for fluid convection, a prototypical chaotic system
- analysis of a model for shear flow turbulence derived using Galerkin projection
- basics of center manifold reduction
- derivation of low-dimensional models using symmetry methods
- analysis of a model for binary fluid convection derived using symmetry methods
- derivation of low-dimensional models using the proper orthogonal decomposition
- analysis of a model for shear flow turbulence derived using the proper orthogonal decomposition
- pseudospectra analysis of linear models for shear flow turbulence

## Prerequisites

No background in neuroscience or fluid dynamics is required. However, a solid preparation in mathematics, including differential equations, integral calculus, and linear algebra is essential, as is some experience in using mathematics to model the real world. Graduate students in engineering, mathematics, and physics, will have such backgrounds. Much of the material is best explored through computer simulations, and problem sets are an important component of the course. Previous experience with computers is not essential, but the student will need to learn useful aspects of MATLAB and/or other programs for scientific computation.

## Reading Materials

There is no single book which covers all of the topics in this course. The following books, which will be useful at various times, have been placed on reserve at the main library. Please check these out at the Circulation Desk.

- L. Edelstein-Keshet, *Mathematical Models in Biology*, QH323.5.E34 1988
- C. Koch, *Biophysics of Computation: Information Processing in Single Neurons*, QP357.5.K63 1999
- J. D. Murray, *Mathematical Biology*, 2nd edition, QH323.5.M88 1993 - Note: there is a newer edition of this book
- L. Glass and M. C. Mackey, *From Clocks to Chaos: The Rhythms of Life*, QH527.G595 1988
- A. T. Winfree, *The Geometry of Biological Time*, QH527.W55 1980 - Note: there is a newer edition of this book

- A. T. Winfree, *The Timing of Biological Clocks*, QH527.W56 1986
- P. Holmes, J. L. Lumley, and G. Berkooz, *Turbulence, Coherent Structures, Dynamical Systems, and Symmetry*, QA913.H65 1996
- S. H. Strogatz, *Nonlinear Dynamics and Chaos: With Applications in Physics, Biology, Chemistry, and Engineering*, Q172.5.C45 S767 1994
- D. C. Hanselman, B. Littlefield *Mastering MATLAB 6: A Comprehensive Tutorial and Reference*, QA297.H293 2001
- N. J. Higham and D. J. Higham, *MATLAB Guide*, QA297.H5217 2000

The lectures will sometimes draw upon material in specific research articles. These articles can typically be downloaded off the web, with website address given on the Course Webpage.

### Reading Materials - Also of Interest

Although the following books are not on reserve - the UCSB library doesn't have them - they also give good treatments of issues related to modeling in neuroscience:

- D. Johnston and S. M-S. Wu, *Foundations of Cellular Neurophysiology*
- J. P. Keener and J. Sneyd, *Mathematical Physiology*
- C. P. Fall, E. S. Marland, J. M. Wagner, and J. J. Tyson, eds., *Computational Cell Biology*
- W. Gerstner and W. Kistler, *Spiking Neuron Models*
- F. C. Hoppensteadt and E. M. Izhikevich, *Weakly Connected Neural Networks*
- A. Pikovsky, M. Rosenblum, and J. Kurths, *Synchronization: A Universal Concept in Nonlinear Sciences*
- S. Strogatz, *Sync: The Emerging Science of Spontaneous Order*

The following books describe dynamical systems theory, which is very useful for low-dimensional modeling:

- J. Guckenheimer and P. Holmes, *Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields*
- S. Wiggins, *Introduction to Applied Nonlinear Dynamical Systems and Chaos*
- P. Glendinning, *Stability, Instability, and Chaos*

## Homework/Grading

- Grades will be determined by performance on several homework sets. These will often involve computer simulations, and the necessary background will be provided in the lectures - see below for computer issues. Assignments will be put on the Course Webpage.

**The homework will be due at the time given on the assignments. Because solution sets will be posted on the Course Webpage shortly after the due date, please get the assignments in on time!**

**All homework must be turned in by the last day of instruction, Friday, March 12!!!**

- NO FINAL EXAM

## Office Hours

Thursdays 1:00-2:00, Room 2350 Engr II Bldg

## Email List

A roster including email addresses will be compiled at the first few lectures so that students can be contacted with important announcements and homework tips.

## Computer Issues

It is encouraged that students use MATLAB for the homework. This is an integrated technical computing environment that combines numeric computation, advanced graphics and visualization, and a high-level programming language. A MATLAB primer is available from the Course Webpage, and several books on MATLAB have been put on reserve in the library.

Another computer program which may be of interest is XPP, a tool for solving differential equations, difference equations, delay equations, functional equations, boundary value problems, and stochastic equations. XPP can be downloaded from

<http://www.math.pitt.edu/~bard/xpp/xpp.html>

Note that this website has a nice tutorial for learning to use XPP; the author of this program, Bard Ermentrout, has also recently published a book called *Simulating, Analyzing, and Animating Dynamical Systems: A Guide to Xppaut for Researchers and Students*, SIAM, 2002.

Students with access to other equivalent programs are welcome to use them to complete the homework sets.