Mathematics of Engineering - ME17
Spring 2008

Notes on Probability

Misunderstanding of probability may be the greatest of all impediments to scientific literacy
- Stephen Jay Gould

These notes cover material that you are responsible for, but which is not covered the textbook.

Let N be the number of trials, and let n be the number of times a particular event occurs in those N trials. The, the probability p of that particular event occurring is defined to be

$$p = \lim_{N \to \infty} \frac{n}{N}. \tag{1}$$

Note that we must have $0 \leq p \leq 1$.

Events are defined to be *mutually exclusive* if only one of them can occur in a given trial. The following rule applies to mutually exclusive events:

Rule 1 If $p_1, p_2, \cdots p_r$ are the separate probabilities of r mutually exclusive events, the probability P that one of these events will occur in a given trial is

$$P = p_1 + p_2 + \cdots + p_r. \tag{2}$$

Events are said to be *independent* if the occurrence of any of them in no way influences the probability of occurrence of any other. The following rule applies to independent events:

Rule 2 If $p_1, p_2, \cdots p_r$ are the separate probabilities of occurrence of r independent events, the probability P that they will all occur in a single trial is

$$P = p_1 \times p_2 \times \cdots \times p_r. \tag{3}$$

The concept of "expectation" has a well-defined definition in the theory of probability. Well, actually the definition depends on whether the variable of interest is discrete or continuous.

Discrete variable case: Suppose that the possible results in a trial are $x_1, x_2, x_3, \cdots$ and their associated probabilities are $p_1, p_2, p_3, \cdots$. Note that we must have

$$\sum_i p_i = 1, \tag{4}$$

where the sum is taken over all possible results. The *expected value* $E(x)$ of x is defined to be

$$E(x) = \sum_i x_i p_i. \tag{5}$$

More generally, the expected value $E(f(x))$ of $f(x)$ is

$$E(f(x)) = \sum_i f(x_i) p_i. \tag{6}$$

An example would be $f(x) = (x - \mu)^2$, where μ is the mean value. Then $E(f(x))$ gives the expected variance.

Continuous variable case: If the possible results in a trial can take any finite value, and the associated probability density function is $p(x)$, then the *expected value* $E(x)$ of x is defined to be

$$E(x) = \int_{-\infty}^{\infty} x p(x) dx. \tag{7}$$

Note that we interpret $p(x)dx$ to be the probability that the result is between x and $x + dx$. Thus, we must have

$$\int_{-\infty}^{\infty} p(x) dx = 1. \tag{8}$$

More generally, the expected value $E(f(x))$ of $f(x)$ is

$$E(f(x)) = \int_{-\infty}^{\infty} f(x) p(x) dx. \tag{9}$$