



# A quasispecies

is a population of RNA or DNA molecules:

ATCAGGACTCA

ATCGGGACTCA

ATCAGGAATCA

....

Manfred Eigen & Peter Schuster 1977

# The Quasispecies equation

Frequency of type  $j$

Mutation from  $j$  to  $i$

$$\dot{x}_i = \sum_{j=1}^n x_j f_j Q_{ji} - f(\bar{x}) x_i$$

Fitness of type  $j$

Average fitness

$$f(\bar{x}) = \sum_i f_i x_i$$

$$\sum_i x_i = 1$$



# The Quasispecies equation

$$\dot{x}_i = \sum_{j=1}^n x_j f_j Q_{ji}$$

$$\dot{x}_i = \sum_{j=1}^n x_j Q_{ji} + f_i x_i$$



# Mutation matrix

$$Q_{ij} = p^{H_{ij}} (1-p)^{L-H_{ij}}$$

$p$ ...mutation rate per bit

$L$ ...genome length

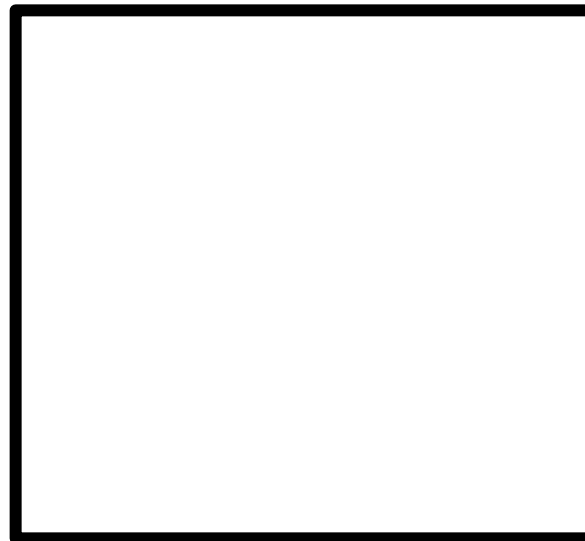
$H_{ij}$ ...Hamming distance

(= number of point mutations  
between  $i$  and  $j$ )

# Sequence space

10

11

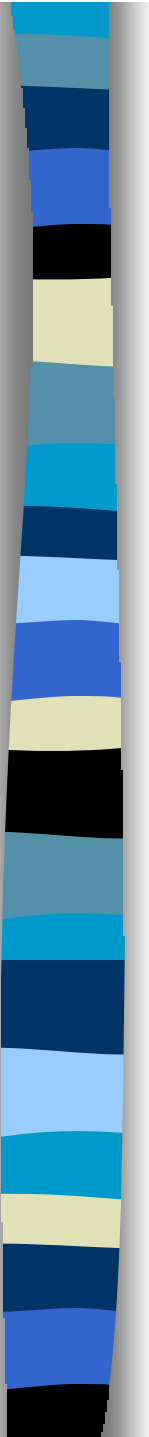


$L=2$  dimensions

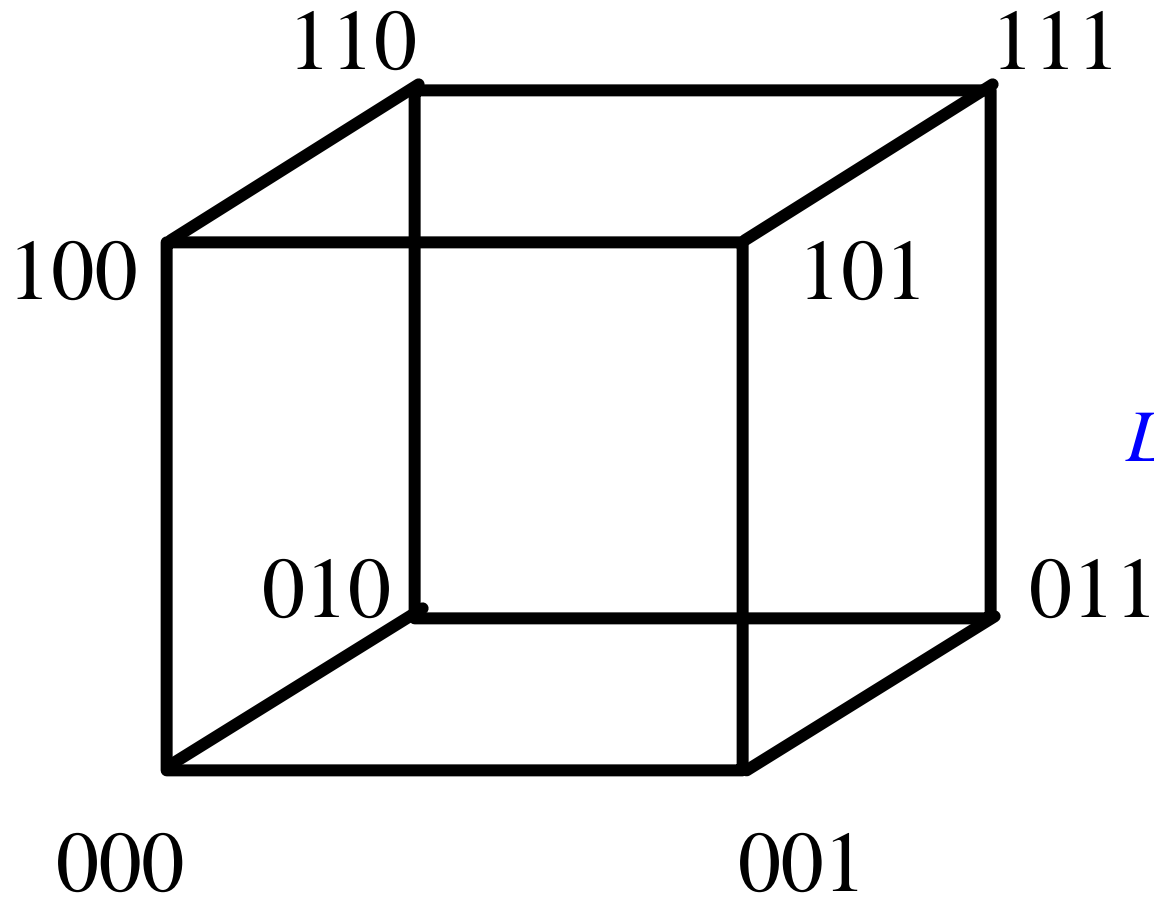
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Hamming (or Manhattan) metric



# Sequence space

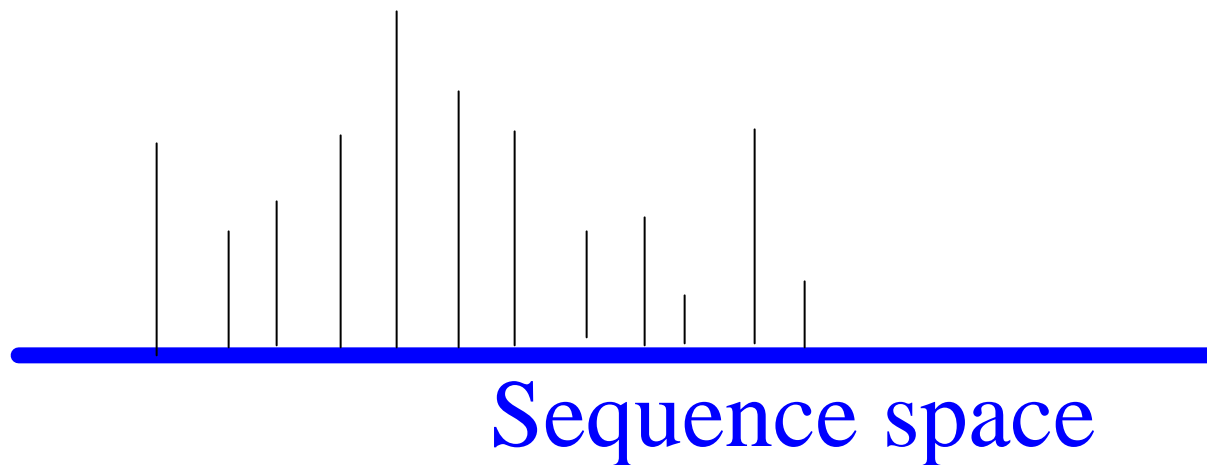


$L=3$  dimensions



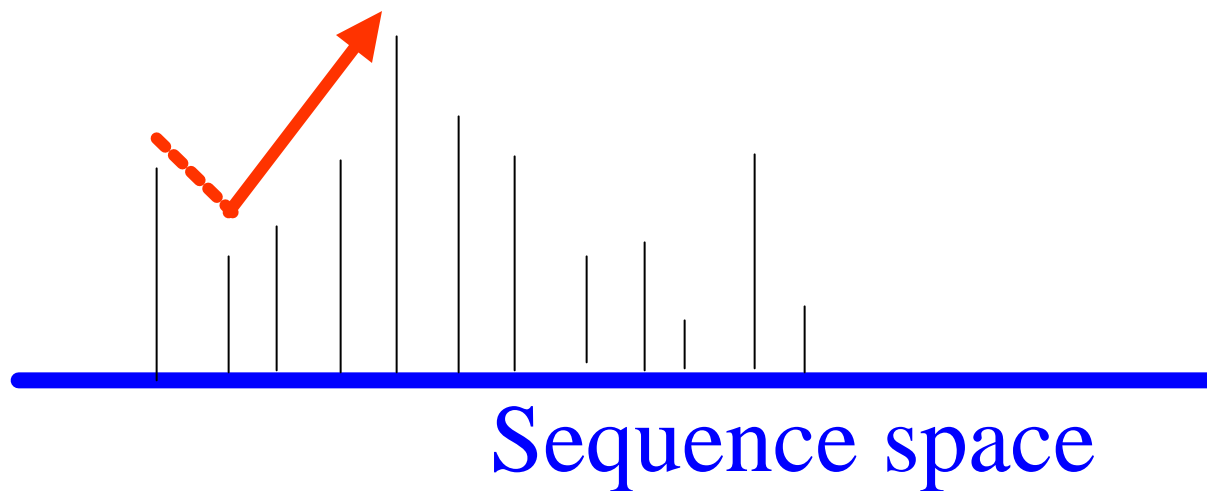
# Fitness landscape

Each point in sequence space is assigned a number.



# Evolution

... is adaptation of the **quasispecies** on the fitness landscape.







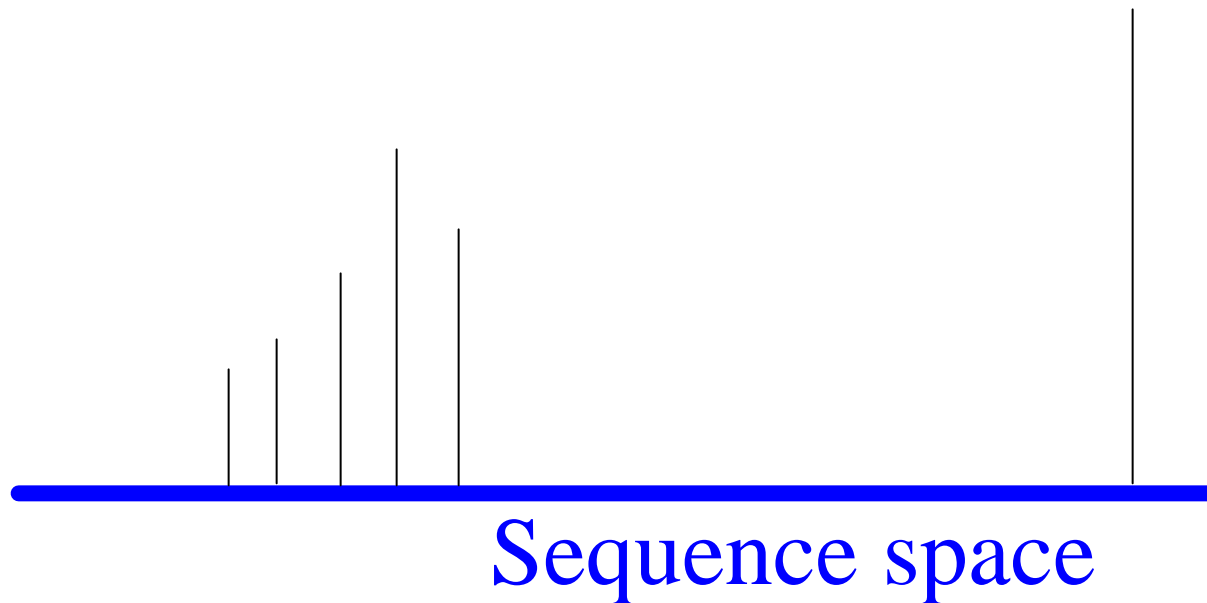
# Search process

Not a single sequence, but the a whole ensemble of sequences explores the fitness landscape.



# Survival of the quasispecies

Not the fittest sequence (maximum  $a$ ),  
but the fittest quasispecies is selected  
(maximum  $\Phi$ ).





# Error threshold

If the mutation rate is too high,  
the quasispecies cannot maintain  
genetic information.  
Advantageous mutants disappear.

# Error threshold



Wildtype

$$\dot{x}_1 = f_1 x_1 Q - x_1 \mathbf{f}$$

Mutants

$$\dot{x}_2 = f_1 x_1 (1 - Q) + f_2 x_2 - x_2 \mathbf{f}$$

Wildtype survives if  $f_1 Q > f_2$



Error threshold implies a  
maximum genome length

$$L < 1 / p$$

↑  
genome length

↑  
mutation rate  
per base



# Error threshold

Adaptation is only possible if the mutation rate is below a certain threshold.

$$p < 1 / L$$

Localization



# Applications of quasispecies theory

- origin of life
- in vitro evolution
- viruses
- bacteria
- cancer



# Extensions

- More complicated mutation operator (insertion, deletion, recombination, etc)
- Stochastic formulation (finite population)