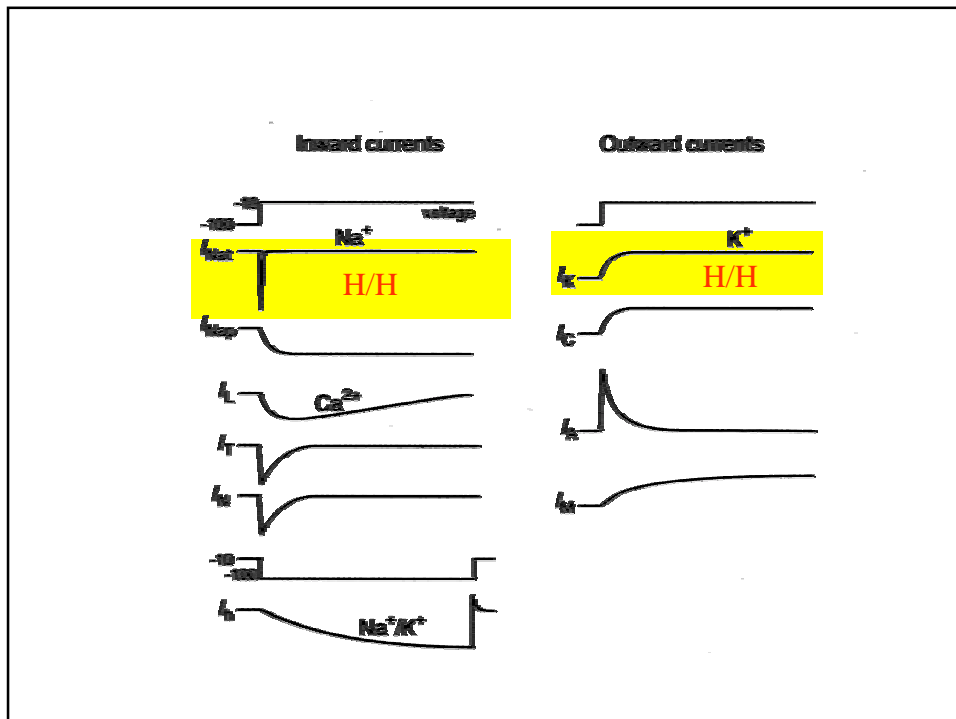


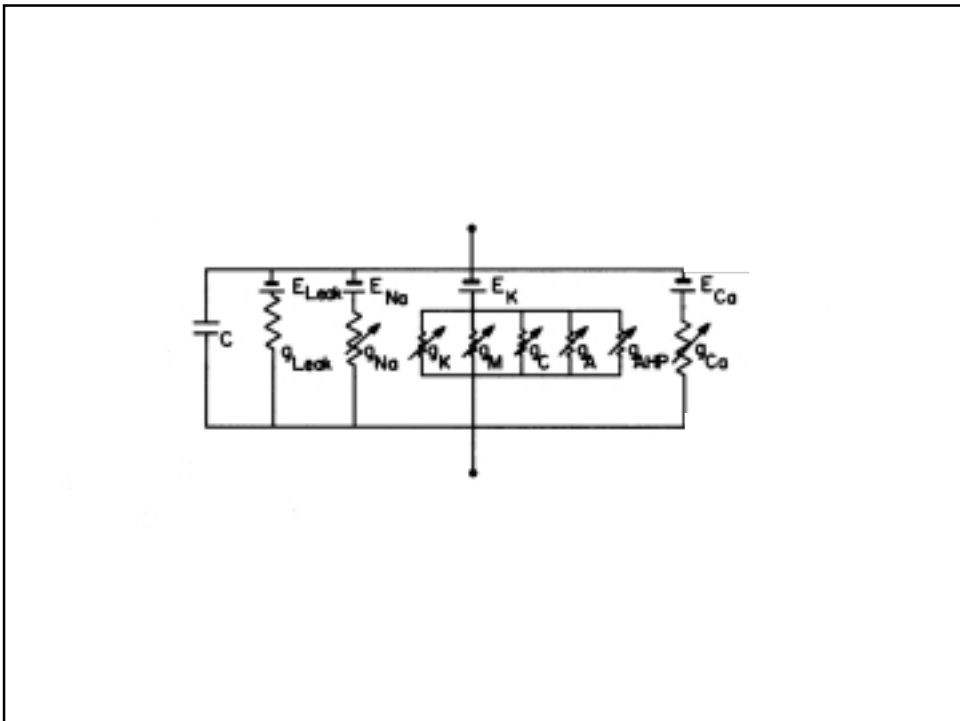
H/H can't generate spikes at low frequency

$$I_A = \bar{g}_{K(A)} m^4 h (V - V_K)$$

Connor & Stevens

m fast, h slow  
turns on quickly following current step and slowly inactivates,  
delaying voltage rise to threshold

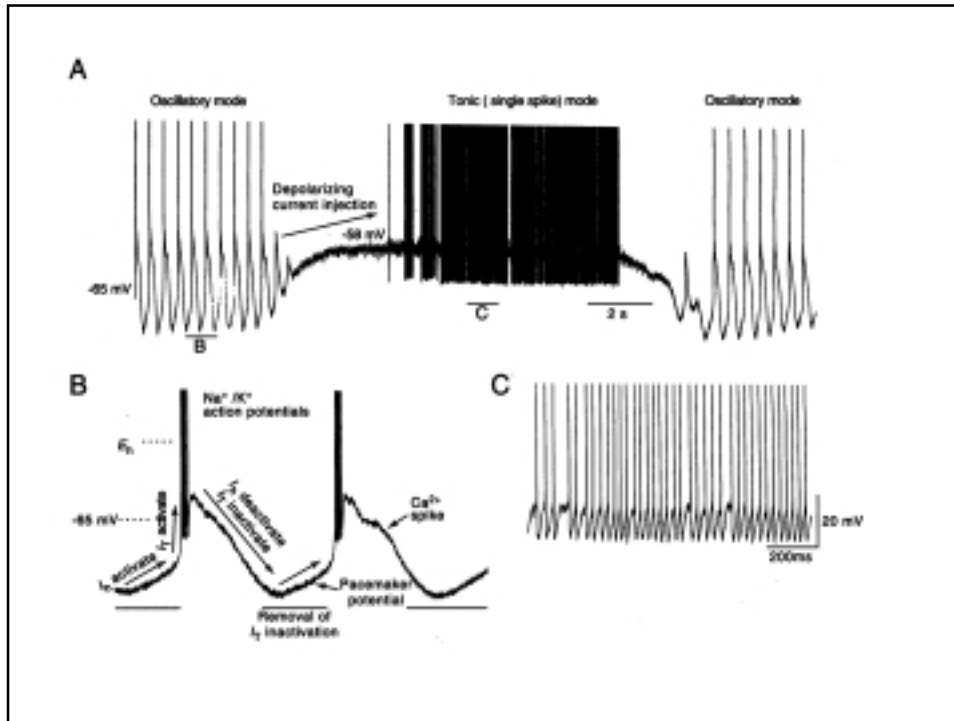




## Thalamocortical Neurons Have Two Firing Modes

Calcium Action Potentials

$I_{Ca(T)}$  “T current” in Bursting



### Intracellular Calcium

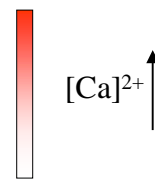
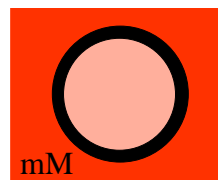
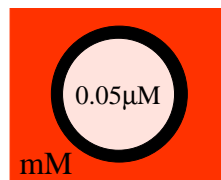
Coupling variable (original H/H) :  $V_m$

New coupling:  $[Ca^{2+}]_{int}$

Calcium-activated potassium current:  $I_{K(Ca)}$

Builds up with each action potential

Decays slowly

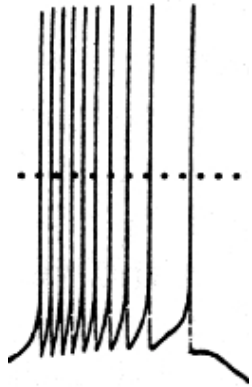


after transmembrane  
calcium ion flux

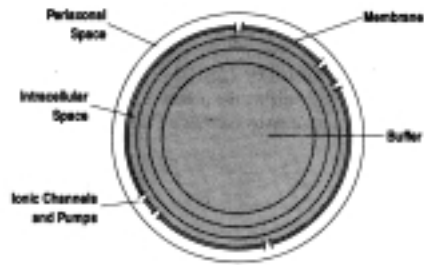
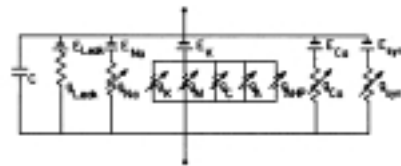
firing rate decays  
with little change  
in  $V_m$

# Adaptation

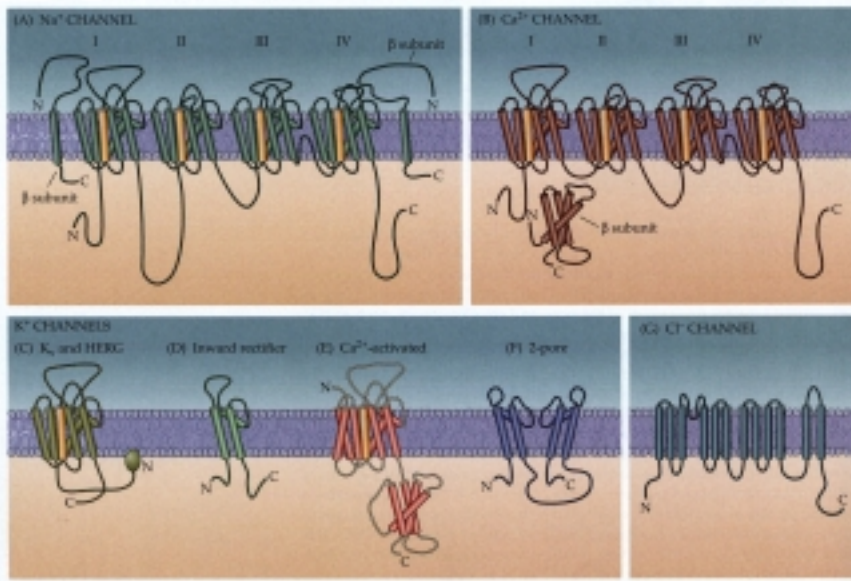
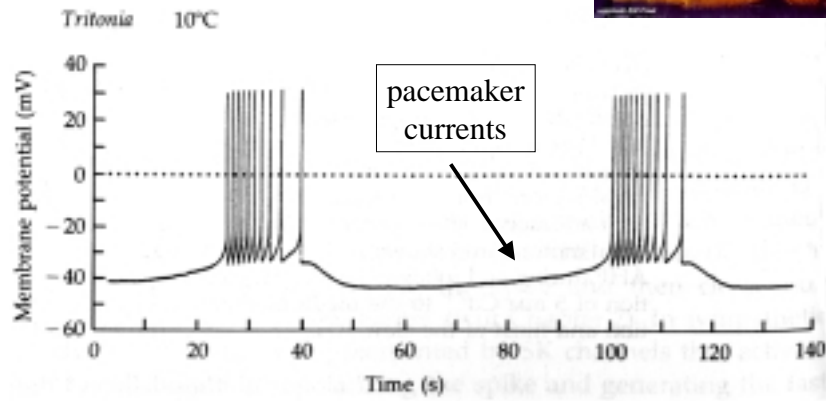
$$I_{K(Ca)}$$

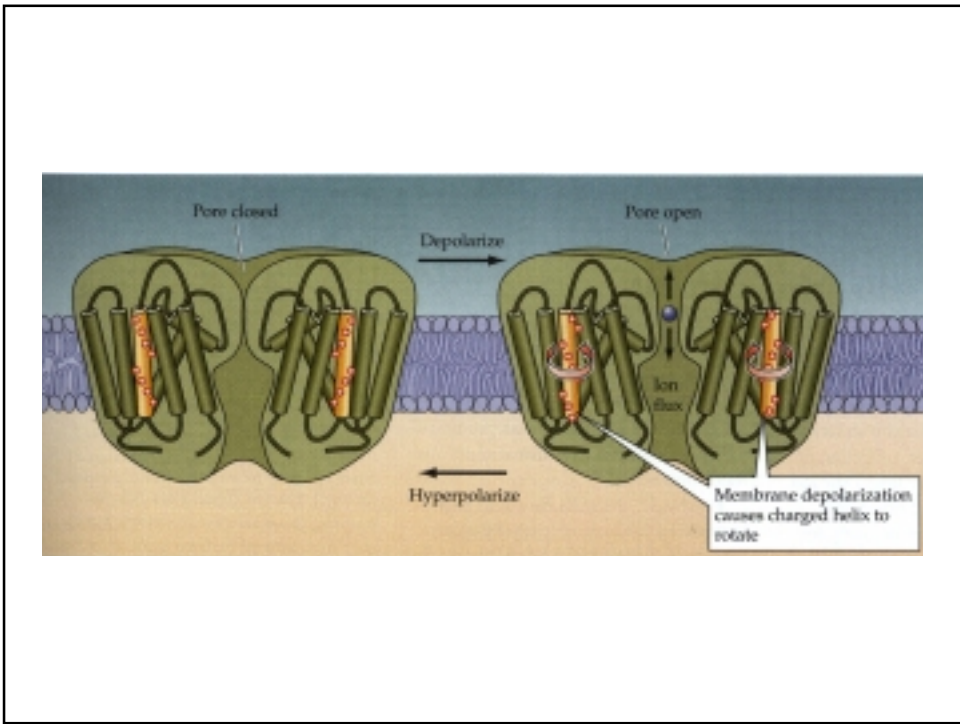
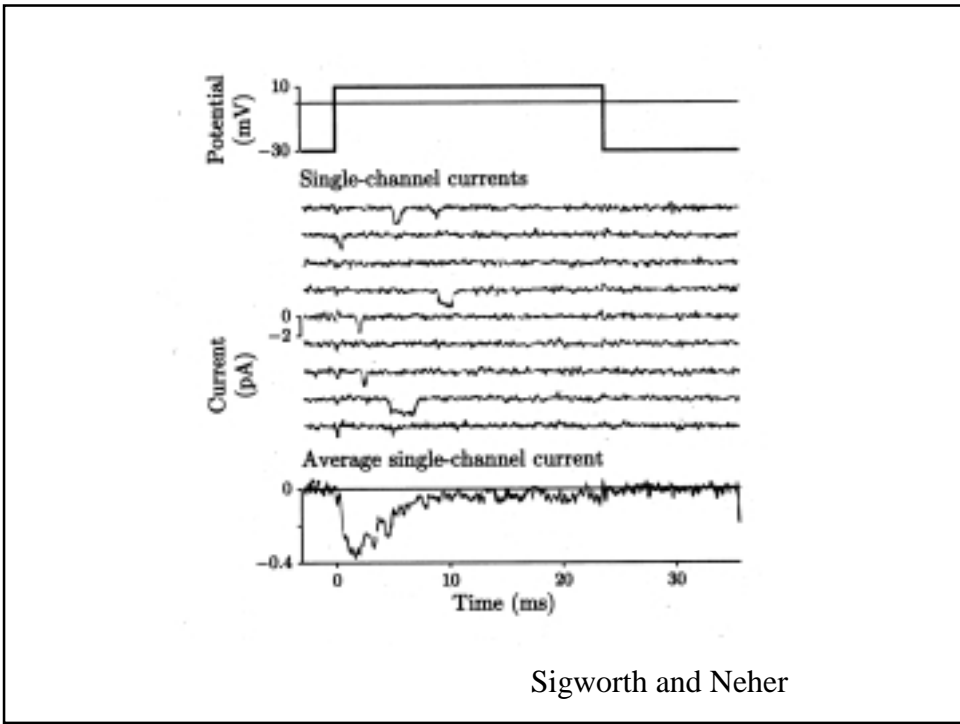


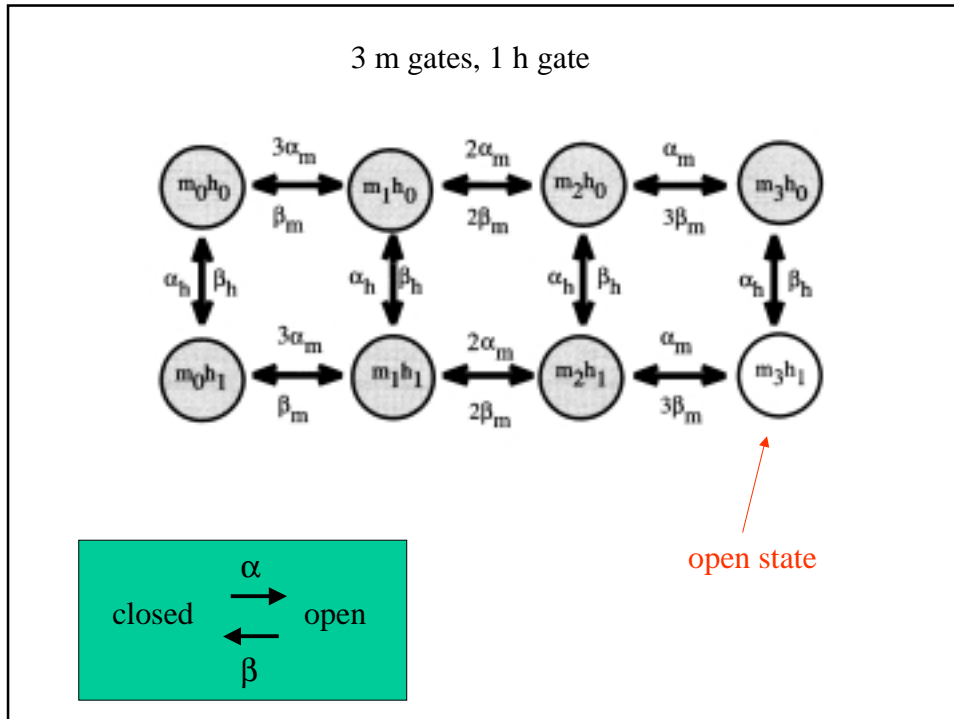
$$I_{K(AHP)} = \bar{g}_{K(AHP)} m([Ca^{2+}]) (V_m - V_K)$$



# Intrinsically Bursting Neuron in CPG







**Monte-Carlo Simulation**

m(t)=probability activation particle in open position  
h(t)=probability inactivation particle in open position

$$P(m_2 h_1 | V_0) = 3 \left( \frac{\alpha_m(V_0)}{\alpha_m(V_0) + \alpha_m(V_0)} \right)^2 \left( 1 - \frac{\alpha_m(V_0)}{\alpha_m(V_0) + \alpha_m(V_0)} \right) \frac{\alpha_h(V_0)}{\alpha_h(V_0) + \alpha_h(V_0)}$$

Compute  $P(m_x h_y | V_0)$  for all states, assign state of individual channel based on random number [0,1] with each state probability a portion of real line.

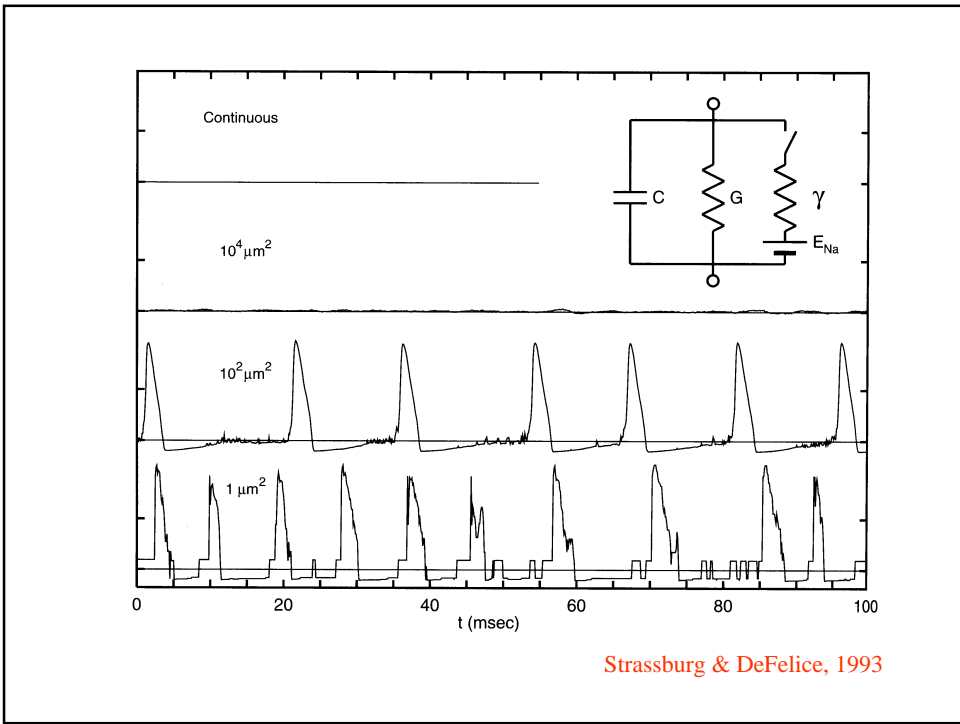
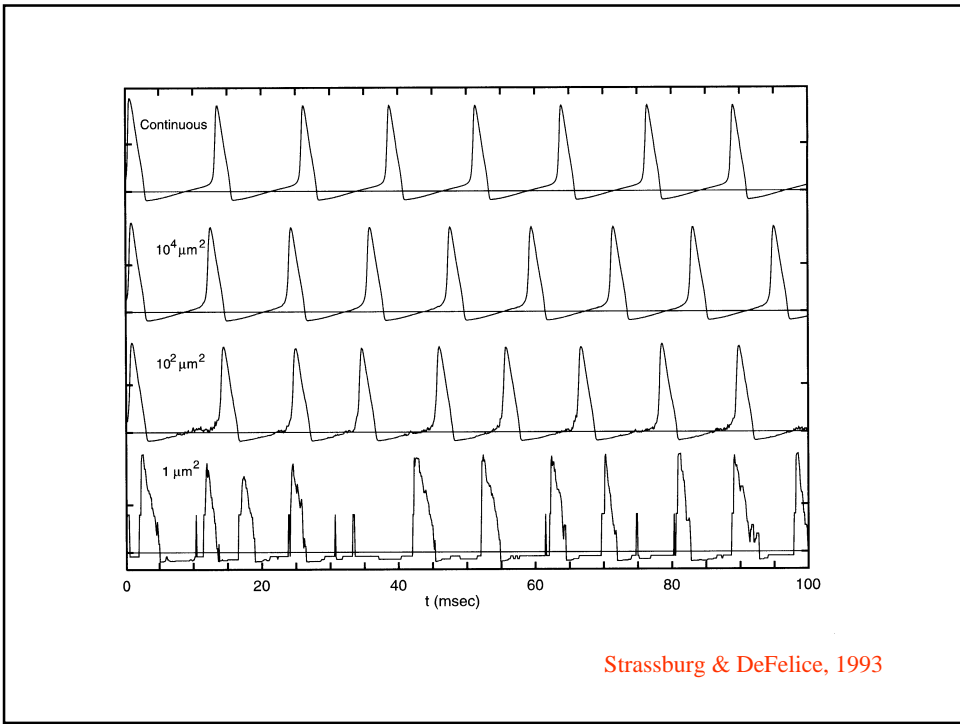
prob  $m_2 h_1$  remain open for T:  $p_2 = e^{-(\alpha_m(V) + \beta_h(V) + 2\beta_m(V))T}$

Random  $p_2 \rightarrow T$

determine which state it goes to:

$$\text{prob}(m_2 h_1 \rightarrow m_3 h_1) = \frac{\alpha_m}{\alpha_m + \beta_h + 2\beta_m}$$

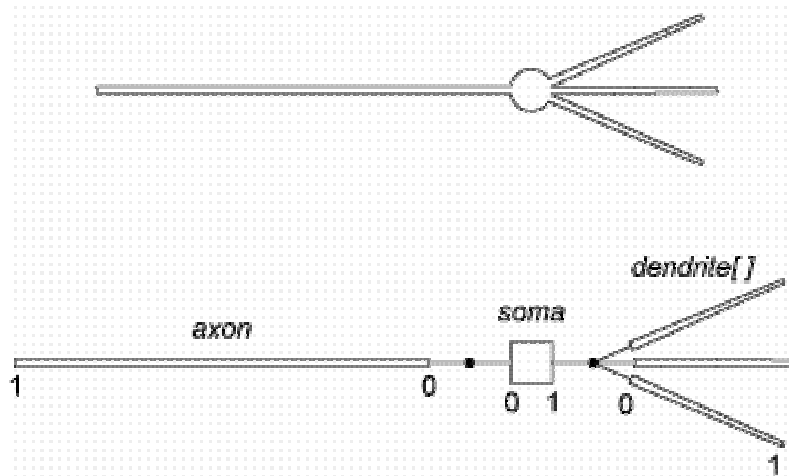
Do transition prob.  
for all states, choose  
random number to  
assign transition



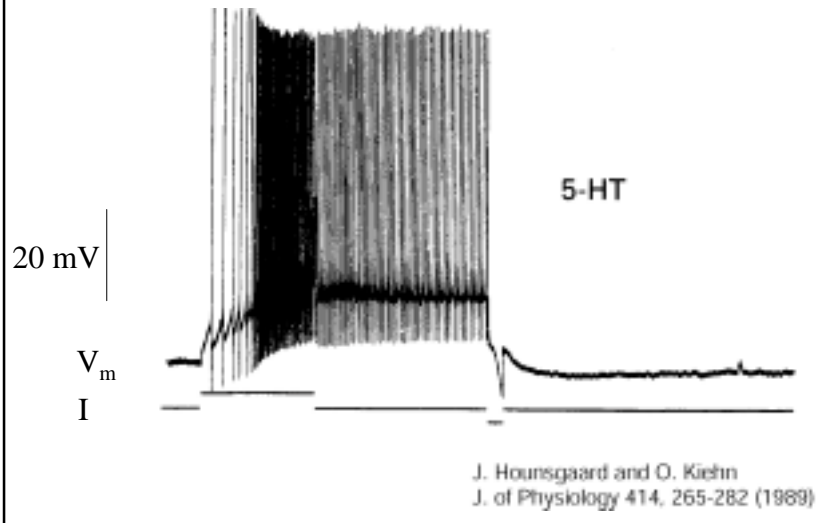


# Neuron Models

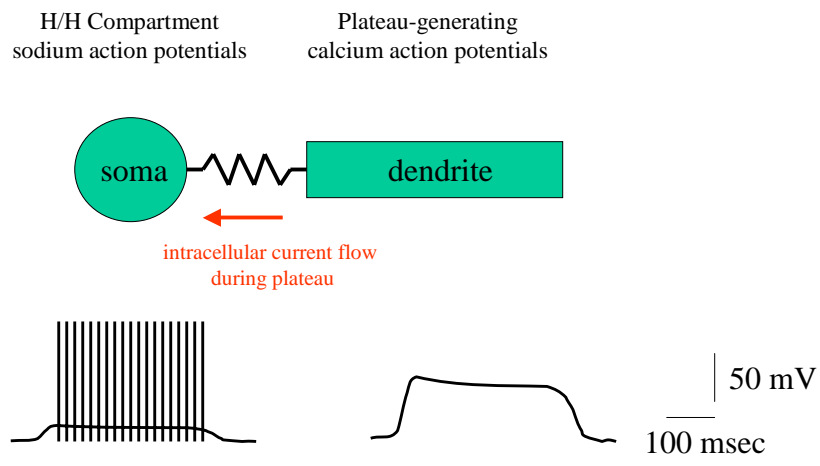
## Compartmental Modeling



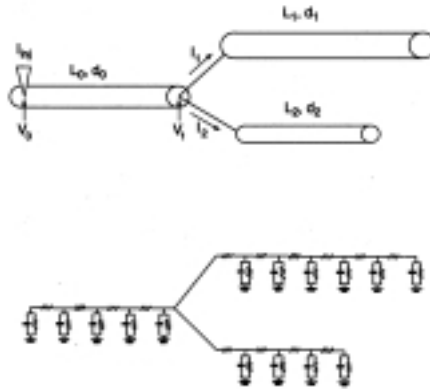
### Plateau Potentials



### Two compartment model of bistable neuron

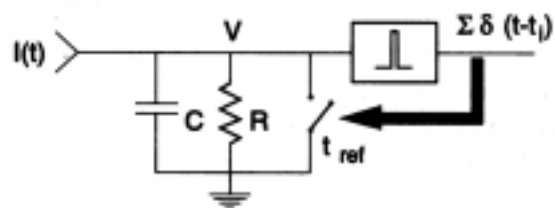


## Compartmental Modeling

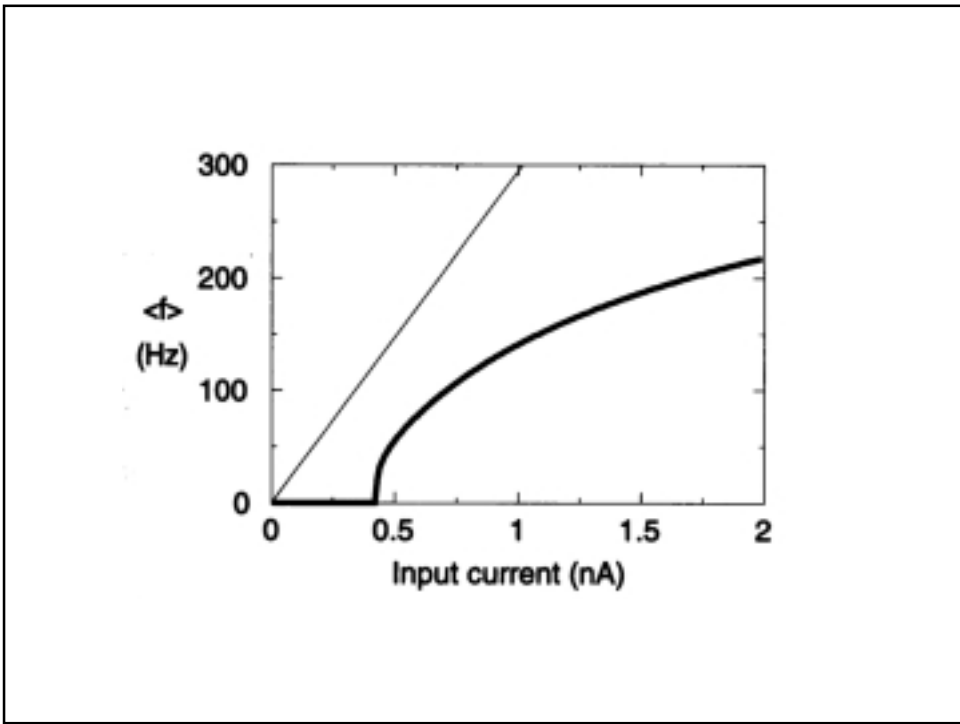
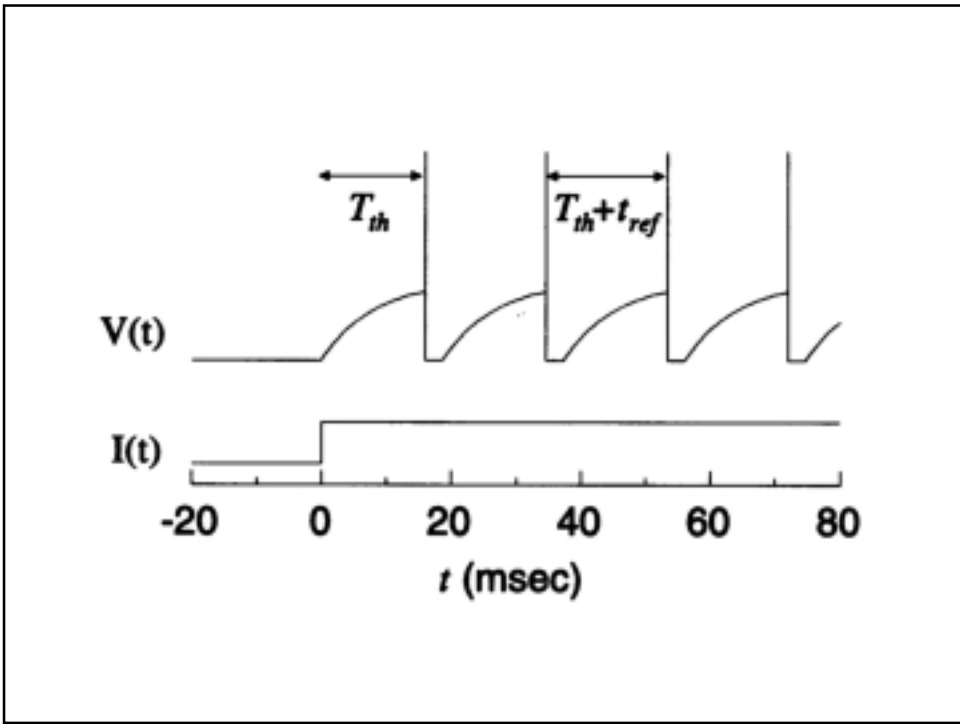


Used primarily for detailed biophysical study of single cells  
Software: **Neuron** (Michael Hines)  
<http://www.neuron.yale.edu>

## Leaky Integrate-and-Fire Unit



$$C \frac{dV}{dt} + \frac{V}{R} = I(t)$$



### Adapting Integrate and Fire Model

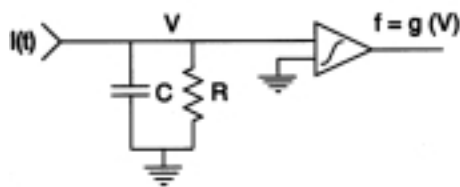
$$C \frac{dV}{dt} = -\frac{V(1 + Rg_{adapt})}{R} + I$$

$$\tau_{adapt} \frac{dg_{adapt}}{dt} = -g_{adapt} + G_{inc} \delta(t)$$

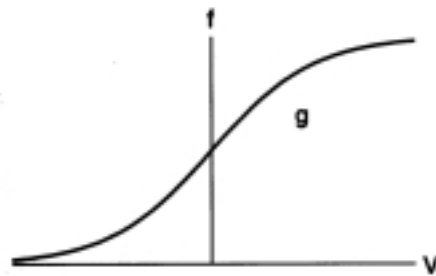
↑  
slow

slow conductance ( $g_{adapt}$ ) with memory in parallel with R

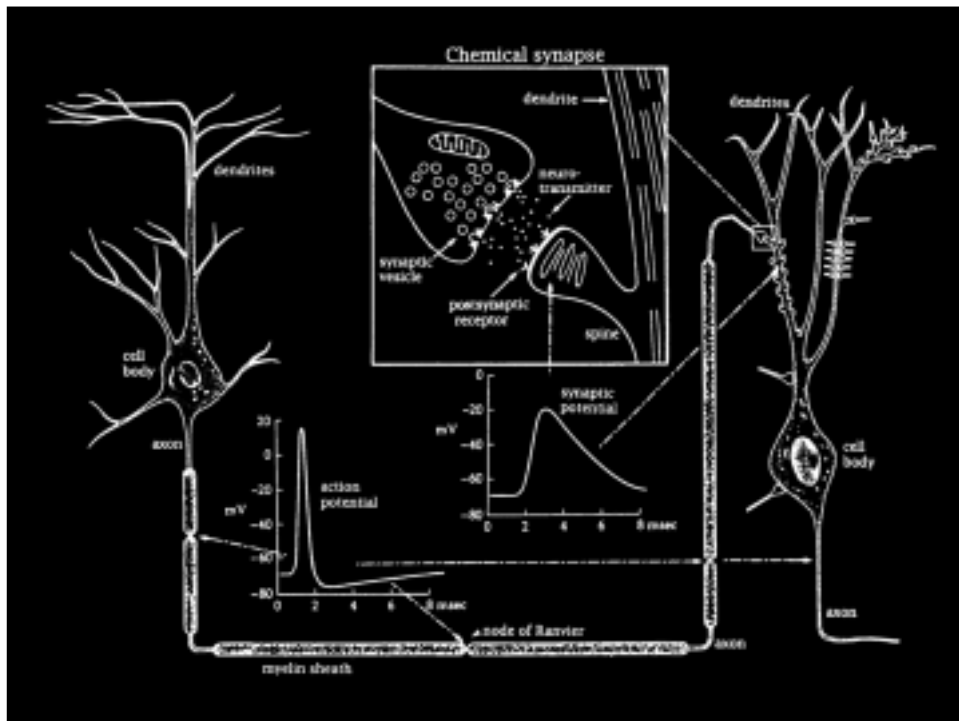
### Firing Rate Model



$$f = \frac{1}{1 + e^{-2\beta V}}$$



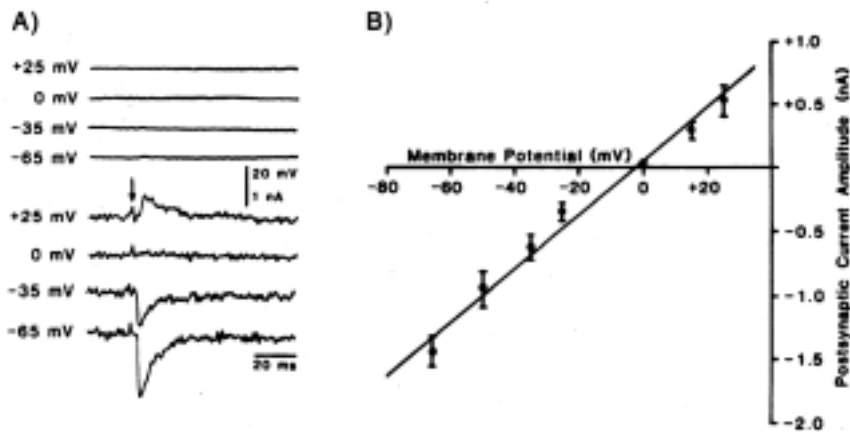
# Synapse/Network Models

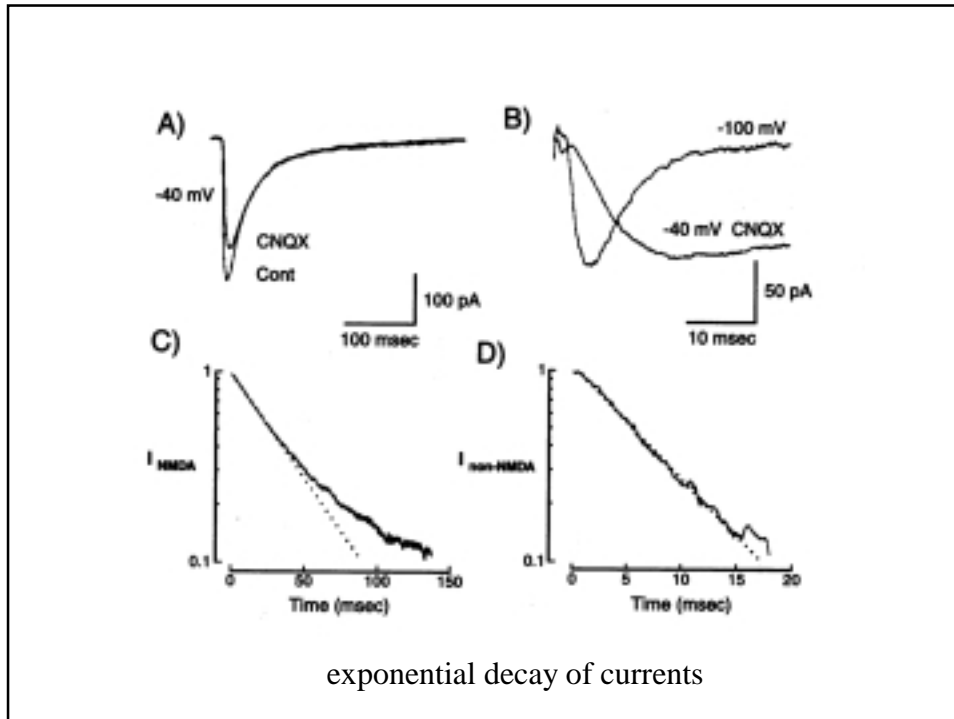


Excitatory  
Inhibitory  
fast/slow

Important concepts for Networks:  
Spatial summation  
Temporal summation

voltage clamp of synaptic current





Augment generalized H/H equations with synaptic conductance terms:

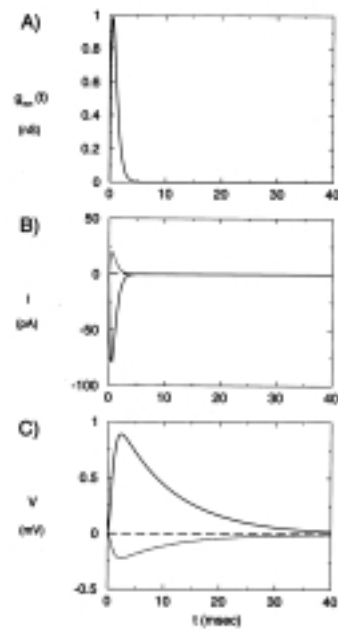
$$I_{syn} = g_{syn}(t)(V_m - V_{syn})$$

$\alpha$  function

$$g_{syn}(t) = \bar{g}_{syn} t e^{-t/t_{peak}}$$

or difference of two exponentials

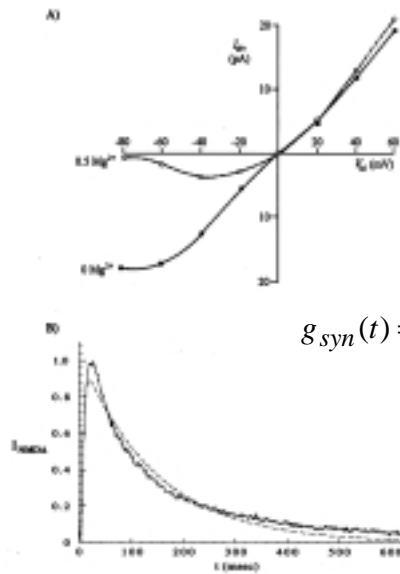
$$g_{syn}(t) = \bar{g}_{syn} (e^{-t/\tau_1} - e^{-t/\tau_2})$$





If  $\Delta V_m$  small, use current pulse:

$$I_{syn}(t) = \bar{I}_{syn} t e^{-t/t_{peak}}$$



conductance  
of NMDA  
receptor synapse

$$g_{syn}(t) = \bar{g}_{syn} \frac{e^{-t/\tau_1} - e^{-t/\tau_2}}{1 + \eta[Mg^{2+}]e^{-\gamma V_m}}$$

$$\tau_1 = 80 \text{ msec}$$

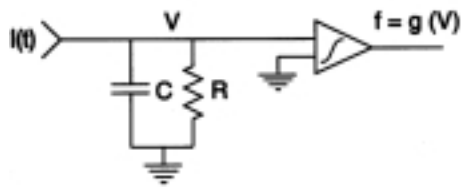
$$\tau_2 = 0.67 \text{ msec}$$

$$\eta = 0.33 / \text{mM}$$

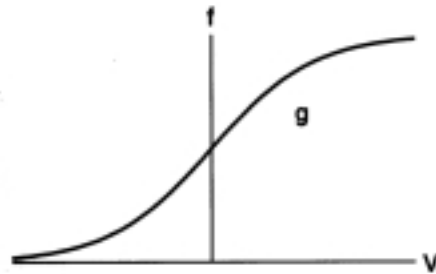
$$\gamma = 0.06 / \text{mV}$$

$$\bar{g}_{syn} = 0.2 - 0.4 \text{ nS}$$

### Firing Rate Model



$$f = \frac{1}{1 + e^{-2\beta V}}$$



### Rate based “neurodynamics”

$$C \frac{dV_i(t)}{dt} = -\frac{V_i(t)}{R} + I_i + \sum_{j=1}^N w_{ij} f_j(t)$$

external input (bias)
↓
↑
synaptic input currents

$w_{ij}$  represents strength of synapse from neuron j to neuron i

$f_i(t) = g(V_i(t))$  input/output relation

assumes linear summation of synaptic inputs  
 precise spike timing ignored

Two neuron firing rate network model

