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Comparison between experiments and numerical simulations of three-dimensional plane wakes

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The three-dimensional structure of a moderate Reynolds number (100) plane wake behind a flat plate subjected to periodic spanwise perturbations has been studied both experimentally and numerically. Comparisons between experimental interface visualizations and numerical calculations demonstrate that important features of the development of the three-dimensional instability can be reproduced by numerical, inviscid vortex dynamics. This suggests a promising way to analyze the physical processes involved in the initiation of the three-dimensional transition in wake flows.

It has long been established that wake flows behind two-dimensional bodies at moderate and high Reynolds numbers can exhibit coherent vortical structures. The formation of these vortical structures, as well as their two- and three-dimensional evolution, is of special interest since it determines not only the drag but also the unsteady forces acting on the body. From a more fundamental point of view, the study of the instability mechanisms resulting in the formation of two- and three-dimensional vortical structures can contribute to our understanding of laminar-turbulent transition and lead to ways to actively control the features of the flow.

Early correlation measurement by Grant[1] showed that the turbulent wake, after initially being dominated by two-dimensional Karman-like vortices, develops strong vortical structures that occur in counterrotating pairs inclined to the plane of the wake. Structures similar to these "vortex pair eddies" or "double roller eddies" were also observed and described by Payne and Lumley,[2] and Mumford,[3] for example. However, at this point, a total understanding of the nature and evolution of the three-dimensional instability mechanism of the wake behind two-dimensional bodies has not yet been achieved.

Recent experimental results (Lasheras *et al.*[4] and Lasheras and Choi[5]) and numerical calculations (Corcos and Lin,[6] Lin and Corcos,[7] and Ashurst and Meiburg[8]) indicate that the origin of the three-dimensional instability in a plane, free shear layer lies in the instability of the strain regions (braids) formed as the primary, two-dimensional, Kelvin–Helmholtz instability develops. In plane wake flows, the primary instability results in the formation of strong two-dimensional vortices of alternating signs, the two-dimensional Karman vortex street [Fig. 1 (a)]. The existence of the strain regions between consecutive two-dimensional vortices suggests that a similar mechanism, in a modified form, might account for the three-dimensional transition.

Inviscid vortex dynamics have been shown to be a reliable modeling technique of the three-dimensional evolutions of plane shear layers.[8] Although the wake initially exhibits two equally strong layers of opposite vorticity versus one dominating sign in the shear layer, it is expected that modeling the wake on the basis of its vorticity dynamics could be an appropriate technique to study this type of shear flows. Thus, in order to analyze the origin and evolution of three-dimensional effects in wakes and to check the suitability of inviscid vortex dynamics for modeling the three-dimensional transition, we have undertaken a combined experimental and numerical study. In this paper, numerical interface calculations via vortex dynamics are compared with flow visualizations of three-dimensional wakes behind a flat plate.

The base flow configuration selected for the study was the two-dimensional wake created behind a flat plate separating two laminar flows of equal velocities. The development of a three-dimensional flow structure in the plane wake was then experimentally investigated by analyzing the response of the two-dimensional base flow to periodic spanwise perturbations of varying magnitude and wavelength. The test rig consisted of a water channel in which two equal velocity, laminar, water streams were produced, initially separated by a flat plate. In the test section, a plane wake was formed as the two parallel streams met at the trailing edge of the splitter plate. (A detailed description of the experimental facility

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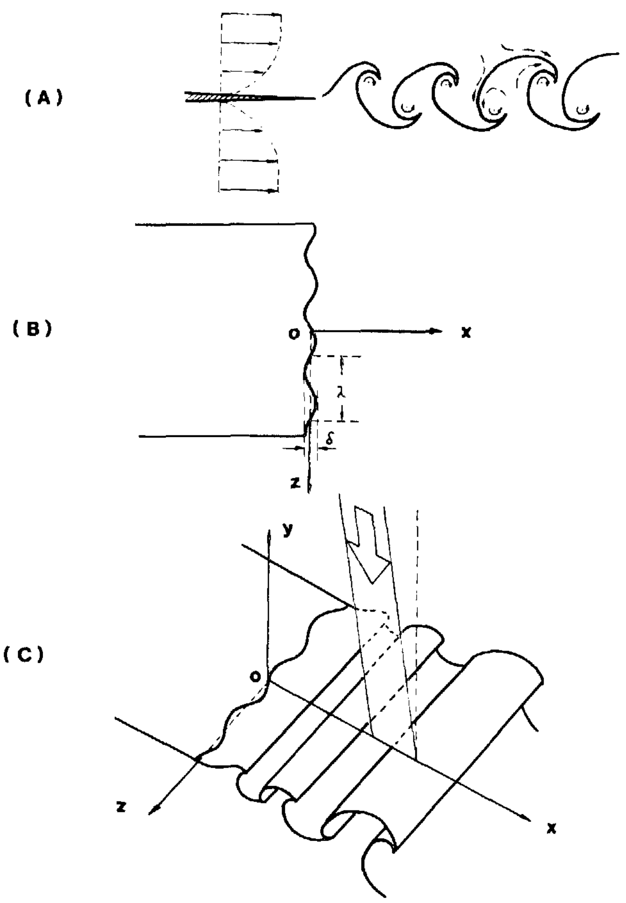

FIG. 1. Plane wake behind a flat plat subjected to sinusoidal perturbations along the span. (a) Side view. (b) Top view of the splitter plate showing the sinusoidal indentation of its trailing edge. (c) View of the wake showing the perspective angle for which interface visualization and numerical calculations were performed.

can be found in Lasheras *et al.*[4]). Periodic spanwise perturbations were introduced by modifying the geometry of the trailing edge of the flat plate from a sharp, straight edge to a sinusoidal shape as shown in Figs. 1(b) and 1(c). To investigate the possible existence of a most unstable wavelength, λ was varied to accommodate from 1 to 10 wavelengths across the span. For each wavelength case, the amplitude δ was always kept small and varied from $\frac{1}{20}$ of the total initial momentum thickness of the wake θ to a maximum of $\frac{1}{10}\,\theta$.

A flow visualization was used to track down the evolution of the interface separating both streams, thus analyzing the development of the three-dimensional instability. The interface was visualized by inducing the fluorescence of fluorescein particles added to the lower stream with a set of spot lights. The spot light arrangement and the fluorescein concentration were selected so that a shadow effect could be created by the corrugated, opaque interface, thereby showing its three dimensionality. To create a solid, opaque effect at the interface, a large concentration of fluorescein particles was needed; however, variations of the physical properties of the water stream caused by the high fluorescein concentration were negligible.

In order to obtain a numerically tractable problem, we limited our calculations to the temporally amplified problem, i.e., the flow field was considered to be periodic both in the span and in the streamwise directions. The nondivergent nature of the velocity field, along with the definition of vorticity, allows for a complete description of the kinematics of the flow in the form of the Biot–Savart law.[9] Using Helmholtz' theorem and following the general concepts reviewed by Leonard,[10] vortex filaments were used for the representation of the vorticity field. To avoid the appearance of singularities, the filaments were modeled having a finite circular cross section, with the vorticity distribution over the core assumed to be algebraic. The position of each filament was represented by a number of nodes through which a cubic spline was fitted to give it a smooth shape. The Biot–Savart integration was then carried out with second-order accuracy both in space and time.

As the flow developed a three-dimensional structure, the vortex filaments underwent considerable stretching. To maintain an adequate resolution, additional nodes were introduced, based on a criterion involving distance and curvature. The filament core radius, following Helmholtz' theorem, decreased as its arclength increased; however, along each filament, the radius was kept at the same value, while the total filament volume remained constant. For the calculations presented here, the wake was modeled with two parallel vorticity layers located at dimensionless transverse coordinates of $+/-0.65$. Both vorticity planes were initially discretized into a total of 150 filaments of diameter 1, each of which was represented by 27 nodes at the beginning of the computation.

The flow visualization used in our experiments enabled us to determine the position of the three-dimensional interface separating both streams and its evolution in space and time. Although the formation of regions of concentrated vorticity can be identified by this method, our visualization does not allow for a detailed measurement of the vorticity field. Thus, for comparison purposes, the position of the interface was also calculated in the numerical simulation. To determine the position of the three-dimensional interface, uniformly spread marker particles were inserted, initially located at the midplane between the two sides of the wake. The trajectories and positions of these particles were then calculated simultaneously with the evolution of the vorticity field. To maintain a good resolution, additional marker particles were introduced as the interface became more stretched by the development of its three dimensionality.

Two perspective views of the experimentally measured (a) and numerically calculated (b) position of the interface of a three-dimensional plane wake are given in Fig. 2. They correspond to the experimental case in which the plane wake was subjected to a periodic spanwise perturbation with a wavelength λ equal to $\frac{2}{3}$ of the most unstable streamwise wavelength λ' [see Fig. 2(a)]. The amplitude of the perturbation δ was $\frac{1}{10}$ of the total initial momentum thickness. For the calculations, the initial disturbance consisted of a wavy dislocation of the filaments' center lines out of the x,z plane, (i.e., in the transverse y direction) identical for both sides of the wake. In the calculations, the imposed spanwise wavelength was $\frac{2}{3}$ of the streamwise wavelength, just as in the

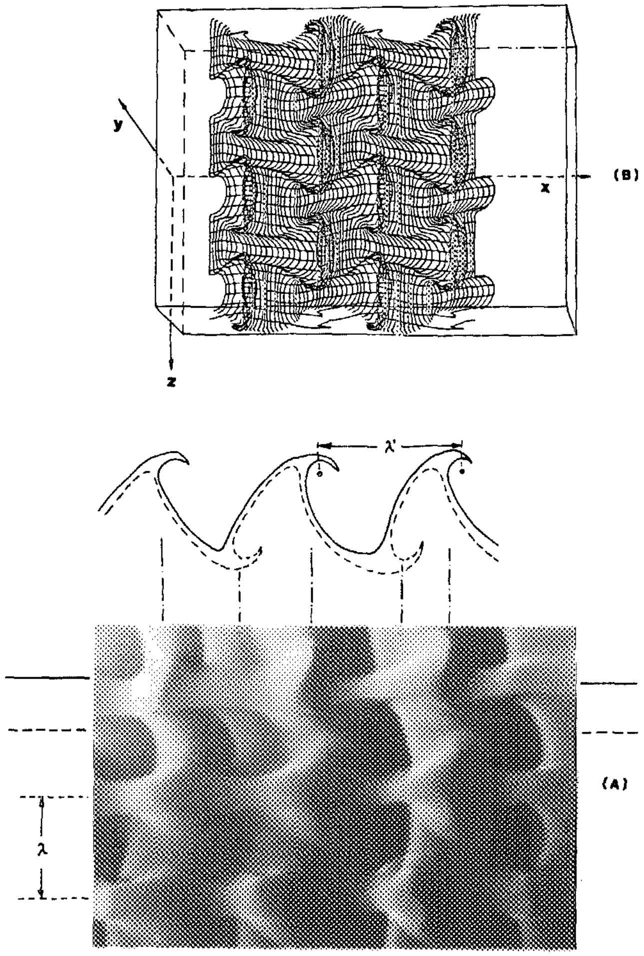

FIG. 2. Near top perspective view. (a) Experimentally visualized and (b) numerically calculated interface. Also shown in the figure are two vertical cross cuts (solid line and dashed line) along planes parallel to x,y.

experiments. Both the experimental and numerical cases correspond approximately to the same distance downstream of the trailing edge of the splitter plate (the same time).

The interface flow visualization (a) and the numerical calculation (b) show the same pattern of three-dimensional deformation of the interface (Fig. 2). In the perspective view, it is clearly visible how the interface wraps around the evolving spanwise Karman-like vortices, while simultaneously it experiences a modulation along the span caused by the appearance of concentrated streamwise vorticity. Under the effect of the strain field created by the spanwise vortices, vorticity concentrates in directions inclined both to the plane of the flow and to the x,y plane, thus resembling the horseshoe vortices observed in plane boundary layers. These pairs of counterrotating streamwise vortices adopt a similar shape to that inferred from correlation measurements by Grant.[1]

The close agreement between the experimental and numerical observations suggest that the techniques outlined above present a promising way for a more detailed analysis of the physical processes governing the initial stages of the three-dimensional transition in wake flows. Simulations based on nearly inviscid vortex dynamics seem to be able to correctly reproduce the mechanisms observed in the experiments and thus provide a powerful tool for studying these flows. A more complete experimental and numerical investigation of plane wakes along these lines is currently underway.

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