

# Three-dimensional features of particle dispersion in a nominally plane mixing layer

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Navier–Stokes simulations of a temporally growing mixing layer are employed to investigate three-dimensional mechanisms for the dispersion and accumulation of small, heavy, spherical particles. It is found that in particular the presence of the streamwise braid vortices gives rise to additional dynamical effects that modify the concentration, dispersion, and suspension patterns observed in two-dimensional situations. Intense stretching and folding by the evolving three-dimensional vorticity field, when combined with inertial effects such as ejection by the concentrated streamwise vortices, strongly distorts the geometry of both clear fluid and particle laden regions. Different time scales can be associated with the spanwise and streamwise vortices, so that these distinct vortical systems can selectively affect different classes of particles. © 1996 American Institute of Physics. [S1070-6631(96)02709-2]

For the last decade, the two-dimensional (2-D) mixing layer has served as the standard test bed for the exploration of the mechanisms by which the large-scale coherent structures of turbulent flows result in the dispersion or accumulation of heavy particles or droplets.<sup>1–9</sup> The ejection of the heavy particles from the vortex cores, and the simultaneous formation of highly concentrated particle bands in the strain field near the braid stagnation point, have been identified as the mechanisms that govern the overall efficiency of the dispersion process. Scaling laws based on local linearizations of the flow field<sup>8,9</sup> have been able to explain the optimal efficiency of these processes for  $St \approx 1$ . A more detailed account is provided by many of the references in the recent overview by Crowe, Troutt, and Chung.<sup>10</sup>

The observed formation of bands of high particle concentrations in the braid region immediately raises the question as to how three-dimensional (3-D) effects can modify the above 2-D picture. Both experimental<sup>11,12</sup> and numerical<sup>13,14</sup> investigations have demonstrated the presence of a system of strained counter-rotating streamwise vortices in this region, which should affect the concentrated particle bands. A first glimpse of the interesting dynamics displayed by heavy particles in an extensionally strained vortex was provided by the model problem of a Burgers vortex.<sup>15</sup> In this flow, particles with a subcritical value of  $St$  can be stably located at the vortex center, whereas supercritical particles tend to orbit around the vortex. An extension of this investigation to the quasi-2-D problem of a periodic array of extensionally strained counter-rotating vortices<sup>16,17</sup> yielded a wealth of information regarding possible accumulation regions and nonlinear particle dynamics in the mixing layer braid region.

The present investigation aims at integrating the features extracted from the above 2-D and quasi-2-D model flows into a coherent picture of 3-D particle dispersion. Toward this end, we analyze the dispersion of heavy particles in a three-dimensionally evolving, temporally growing mixing layer. The assumption of dilute particle concentration allows us to neglect the effect of the particles on the fluid motion, so that the fluid velocity can be obtained in the usual way by

means of direct numerical simulation of the Navier–Stokes equations, e.g., Ref. 18. The flow field is periodic both in the streamwise and the spanwise directions. The Reynolds number, formed with half the difference velocity and the initial vorticity thickness, has a value of 200. Two-dimensional initial disturbances in the form of the most unstable eigenfunctions are combined with periodic streamwise vortical perturbations in order to trigger an evolution that approximately reproduces the ROLLUP case analyzed in detail in Ref. 18. The comparison with this case simultaneously serves as validation for our numerical code. The flow is characterized by initially weak counter-rotating braid vortices, which amplify under the action of the spanwise Kelvin–Helmholtz vortices until they reach a critical strength that leads to their collapse into nearly round concentrated vortex tubes. The effect of vortex pairing is not considered within the present investigation.

At the beginning of the simulation, the upper stream is seeded with 65 000 heavy particles, whose initial velocity is equal to the local fluid velocity. Subsequently, the motion of these particles is calculated by balancing their inertia with gravitational forces and the Stokes drag exerted on them by the fluid. The resulting dimensionless equation of motion for the particle velocity  $\mathbf{v}_p$  as a function of the fluid velocity field  $\mathbf{u}(\mathbf{x}, t)$  and time  $t$

$$d\mathbf{v}_p/dt = (1/St)[\mathbf{u}(\mathbf{x}_p, t) - \mathbf{v}_p(t)] + (1/Fr^2)\mathbf{e}_g \quad (1)$$

arises from the well-known Maxey–Riley equation<sup>19</sup> in the limit of large density ratio.<sup>2</sup> Here,

$$St = (d^2\rho_p U_0/18\rho_f\nu\delta_\omega^0), \quad Fr = U_0/\sqrt{\delta_\omega^0 g} \quad (2)$$

denote the Stokes and Froude numbers, respectively.  $\rho_f$  and  $\rho_p$  indicate the densities of the fluid and the particle respectively,  $d$  is the particle diameter,  $U_0$  represents half the velocity difference between the free streams,  $\delta_\omega^0$  indicates the initial vorticity thickness,  $\nu$  is the fluid kinematic viscosity, and  $g$  denotes the gravitational acceleration in the direction  $\mathbf{e}_g$ . The particle motion in the streamwise  $x$ -, the spanwise  $y$ -, and the transverse  $z$  direction is obtained by numerically

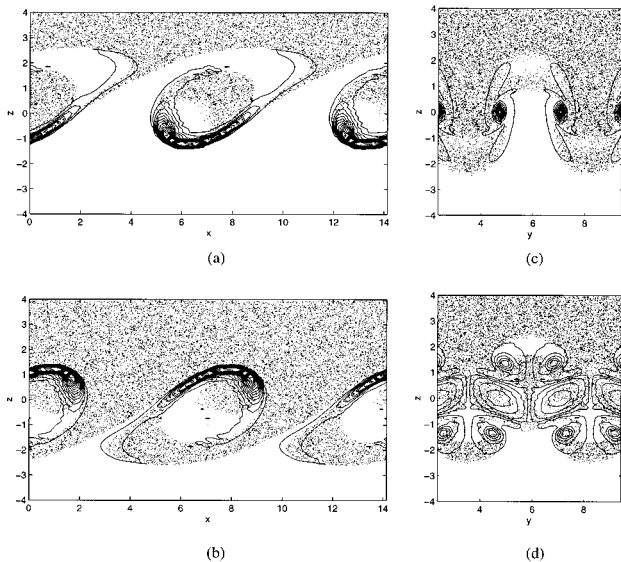


FIG. 1. (a)  $x,z$  cut through the upwelling region at  $t=15$  for  $St=0.01$  in the absence of gravity. (b)  $x,z$  cut through the downwelling region. (c)  $y,z$  cut through the center of the braid region. (d)  $y,z$ -cut through a spanwise roller.

integrating the above equation with a standard predictor-corrector numerical method. For all of the situations reported here,  $\mathbf{e}_g$  points in the  $-z$  direction.

Figure 1 illustrates the case of  $St=0.01$  in the absence of gravity. For this small value of  $St$ , the particles tend to follow the fluid motion, so that even at later times they represent nearly passive markers of the initially seeded stream. Figures 1(a) and 1(b) present  $x,z$  cuts at time  $t=15$  through the upwelling and downwelling regions, respectively, that are created by the streamwise vortical structures. Clearly, large numbers of particles are transported across the central plane of the mixing layer where the braid vortices induce a corresponding component of the fluid motion. Conversely, clear fluid is brought into the initially seeded stream at spanwise locations half a wavelength away. This is a manifestation of how the temporally evolving vorticity field results in complex, 3-D stretching and folding of the initially 2-D interface separating the particle laden and clear fluid regions. These 3-D deformations are well known from single phase flows<sup>11,12,14</sup> and result in the familiar mushroom shapes often observed by means of laser-induced fluorescence. Figures 1(c) and 1(d) show such mushroom patterns in  $y,z$  cuts of the particle concentration field at  $t=15$ . Figure 1(c) represents a cut through the center of the braid region, where the streamwise vortices have the form of concentrated round vortex tubes. Figure 1(d), on the other hand, displays a  $y,z$  cut through a Kelvin–Helmholtz vortex sandwiched between counter-rotating streamwise vortical structures. For  $St=0.01$ , the particle velocity field is nearly divergence free, so that particle accumulation or concentration in pronounced bands is not observed. It should be mentioned that the islands of clear fluid visible in Figs. 1(c) and 1(d) are not due to an ejection of particles from the cores of the streamwise vortices, but rather to the 3-D stretching and folding of the interface between clear and particle laden fluid alluded to above.

Figure 2 illuminates the effect of increased particle inertia for  $St=1$  at the same time and identical locations as in Fig. 1. While the spatio-temporal evolution of the fluid ve-

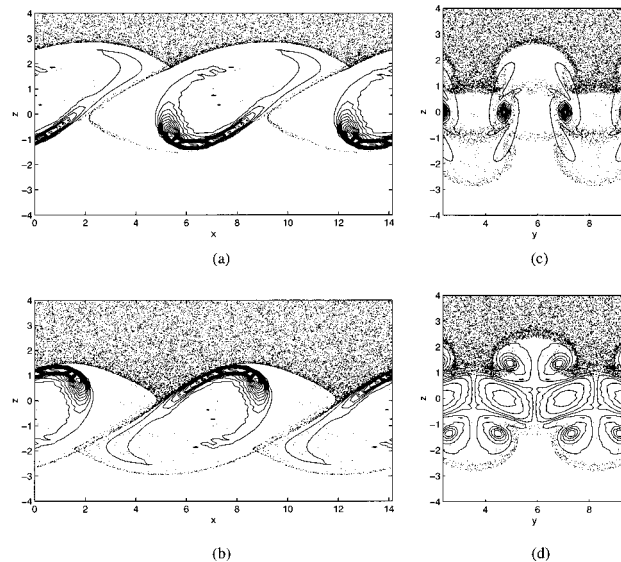


FIG. 2. Cuts at the same time and identical locations as in Fig. 1, for  $St=1$ , without gravity.

locity field is identical to the previous case, the particle concentration field now develops quite differently. Figures 2(a) and 2(b) again demonstrate a clear distinction between transverse particle transport at different spanwise locations, now however in the form of bands of high particle concentrations. These bands, caused by near optimal ejection from the spanwise vortices, can be observed both in the braid region, but also above and below the Kelvin–Helmholtz rollers. Figures 2(c) and 2(d) indicate regions of high particle concentrations in  $y,z$  cuts as well. These “necklace” patterns form partly as a result of the stretching and folding mechanisms observed in Fig. 1. In addition, however, the dynamical interplay between ejection from the streamwise vortex cores and compression due to the strain induced by the spanwise rollers promotes the formation of well-defined periodic particle trajectories, as shown for our earlier quasi-2-D model flow.<sup>17</sup> This tendency is reflected in Figs. 2(c) and 2(d), which demonstrate how the presence of the concentrated streamwise braid vortices contributes to the generation and deformation of both high concentration and clear fluid regions.

Figure 3 demonstrates the effect of gravity on small  $St$  particles, again for the same time and identical locations as in the previous figures. In this case, particles of  $Fr=0.185$  were initially seeded between  $z=0$  and  $z=4$ . They subsequently settle through the mixing layer with a particle velocity field that is approximately equal to the local fluid velocity plus the terminal settling velocity  $St/Fr^2$ , cf. Ref. 20. Since the particle velocity field again is nearly divergence free, regions of high particle concentrations do not form, and ejection from vortex cores does not take place. However, even for this case of strong gravitational effects significant variations of the settling process in the spanwise direction are observed, cf. Figs. 3(a) and 3(b). Figure 3(c) indicates that the settling process is slowed down considerably in the upwelling regions, while Fig. 3(d) shows some particles suspended in the streamwise vortices above the Kelvin–Helmholtz rollers.

For  $St=1$  and  $Fr=1.9$ , Fig. 4 demonstrates the forma-

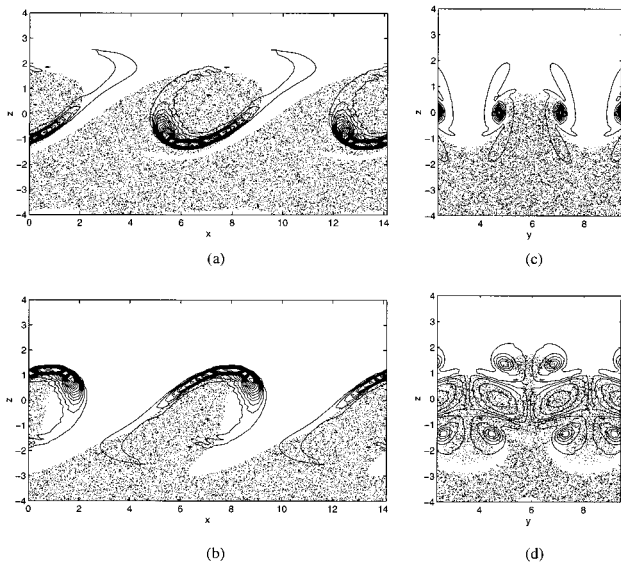


FIG. 3. Identical cuts for  $St=0.01$  and  $Fr=0.185$ .

tion of concentrated particle bands that settle into the lower stream. This is qualitatively similar to the 2-D case,<sup>9</sup> although now the streamwise vorticity results in strong variations in the spanwise direction, cf. Figs. 4(a) and 4(b). The  $y, z$  cuts of Figs. 4(c) and 4(d) show the formation of highly concentrated particle regions connecting the vortex cores, in a fashion that is qualitatively quite similar to the quasi-2-D problem studied earlier.<sup>17</sup> In addition, these figures indicate a strong accumulation of suspended particles in the upwelling regions somewhat below the level of the braid vortices. This behavior, which is in agreement with the accumulation criterion derived earlier for the model flow,<sup>17</sup> is caused by the local convergence of the directional field associated with the spanwise and transverse velocity components.

In summary, 3-D flow effects, and in particular the presence of extensionally strained streamwise braid vortices, give rise to additional dynamical effects that can signifi-

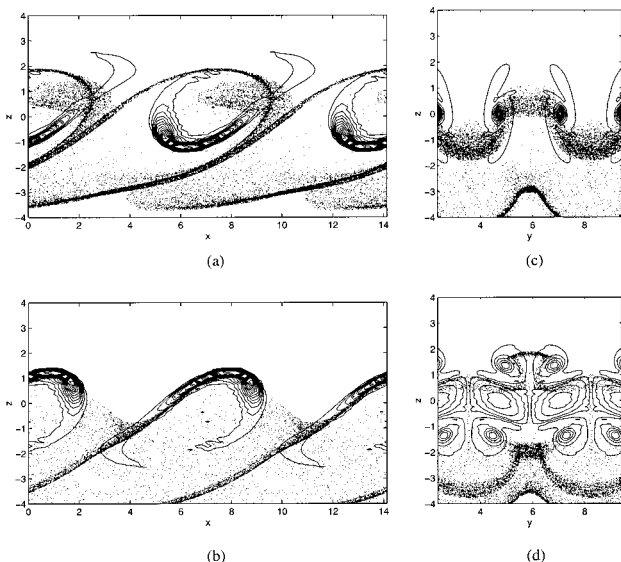


FIG. 4. Identical cuts for  $St=1$  and  $Fr=1.9$ .

cantly alter the concentration, dispersion, and suspension patterns observed in 2-D situations. Increased stretching and folding by the time-dependent 3-D vorticity field, in conjunction with such effects as ejection by concentrated streamwise vortices, lead to a significantly more complex geometry of both clear fluid and particle laden regions. In this context, it is interesting to note that different time scales can be associated with the spanwise and streamwise vortices. As a result, the strongest effects of these different vortical structures might be felt by different classes of particles.

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