

Report of the Symposium on Interactions for Dispersed Systems in Newtonian and Viscoelastic Fluids, Guanajuato, Mexico, 2006^{a)}

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This report summarizes the issues discussed during a Symposium of the International Union of Theoretical and Applied Mechanics, entitled “Interactions for Dispersed Systems in Newtonian and Viscoelastic Fluids,” which was held in March 2006 in Guanajuato, Mexico. © 2006 American Institute of Physics. [DOI: [10.1063/1.2396902](https://doi.org/10.1063/1.2396902)]

I. PREAMBLE

Dispersed systems include drops, bubbles, and particles immersed in a second fluid. The mechanics of such systems is fundamental to the understanding of the flow of multi-phase systems and as a result, strong communities have developed around the respective areas of (i) suspension mechanics, (ii) granular media, (iii) fluidization and suspensions, (iv) interfacial fluid mechanics related to drops and bubbles, and (v) effective medium theory. Such systems are of importance in a variety of technologies, ranging from very large scale processing to the new emerging areas of microelectromechanical systems and microfluidics. Although there have been significant advances in these respective areas, progress in one has often been accomplished without knowledge of analogous advances in another. International scientific meetings permit significant interchange among researchers in these respective fields and promote cross-fertilization.

The International Union of Theoretical and Applied Mechanics (IUTAM) approved a scientific symposium to be held in Guanajuato, Mexico in March 2006. The general theme of the symposium was built around the basic mechanics of drops, bubbles, and particles. In accord with the

IUTAM guidelines, attendance was by invitation, with an emphasis given to involving younger researchers. This was, to our knowledge, the first IUTAM Symposium to be held in Mexico, providing opportunities for interactions and exchanges with the Mexican fluid mechanics community. In addition to the support from IUTAM, partial funding for the Symposium was provided by the Instituto de Investigaciones en Materiales (UNAM), Consejo Nacional de Ciencia y Tecnología (CONACyT-México), the National Science Foundation (NSF-USA), the Academy of Sciences for the Developing World (TWAS) and by the División de Fluidos y Plasmas, Sociedad Mexicana de Física.

The scientific committee for the symposium included George M. Homsy, chair (University of California at Santa Barbara, U.S.A.), Roberto Zenit, co-chair (Universidad Nacional Autónoma de México, Mexico), E. John Hinch (DAMTP, Cambridge University, U.K.), Tsutomu Kambe (University of Tokyo, Japan), L. Gary Leal (University of California at Santa Barbara, U.S.A.), Detlef Lohse (University of Twente, The Netherlands), Jacques Magnaudet (Institute de Mécanique des Fluides de Toulouse, France) and Eduardo Ramos (Universidad Nacional Autónoma de México, Mexico).

The Symposium was focused on the following subjects:

- Particles in Complex Fluids
- Particle-Particle and Particle-Wall Interactions
- Granular Media with Interstitial Fluid Effects
- Bubbles and Drops in Complex Liquids
- Drops, Bubbles, and Particles in Microfluidics

^{a)}With a preamble by Roberto Zenit (Instituto de Investigaciones en Materiales, Universidad Nacional Autónoma de México, Apdo. Postal 70-360 México D.F. 04510, zenit@servidor.unam.mx) and George M. Homsy (Department of Mechanical and Environmental Engineering, University of California, Santa Barbara, Santa Barbara, CA 93106-5070, bud@engineering.ucsb.edu).

- Particles at Interfaces
- Sedimentation

Each session started with a lead talk followed by three or four contributed talks, resulting in a total of seven invited talks and 32 contributed talks. Approximately 40 international and 20 Mexican participants attended. This report, and a number of papers in this special topic section of *Physics of Fluids*, comprises the proceedings. A complete book of abstracts has been deposited with *Physics of Fluids* as supplemental material; it is available through AIP's Electronic Physics Auxiliary Publication Service (EPAPS).¹

The report is organized around the several subjects of the Symposium. The authors have endeavored to interpret and extract the essence of the presentations. We have also included a very current bibliography should readers wish to consult the original sources in more detail.

II. SEDIMENTATION AND SUSPENSION FLOWS

This session was devoted to suspensions in sedimentation and shear flow. Elisabeth Guazzelli (Laboratoire IUSTI, Marseille, France) led off the session with a keynote lecture describing her laboratory's experimental studies of sedimentation of both spheres and slender fibers in a Newtonian liquid. After presentation of background on the hindered settling observed in suspensions and the early theories to address this issue, Guazzelli focused upon the issue of velocity fluctuations and the appearance of viscous vortices in sedimentation of spheres, beginning with an overview of theories presented to describe the fluctuations. Among the results presented were data showing that the velocity fluctuations decay from a maximum at the beginning of the experiment to a fixed value, while the length scale of vortices decay from the box size to a size of approximately 20 mean interparticle spacings ($20a\phi^{-1/3}$, with a the particle diameter and ϕ the solid volume fraction).² The role of stratification of particle fraction was, according to the experiments of this group, found to account for behavior only near the sedimentation front. In the sedimentation of fibers, an enhancement of sedimentation speed relative to the maximum velocity, associated with the fibers settling with their long axis vertical, is seen for low fiber fractions, with hindrance at higher loadings; this is argued to be a result of the fibers forming transient "packets" that settle more rapidly than single fibers.³ The formation of packets is not well understood, but is believed to be related to orientation of the fibers in the shear flow induced by nonuniform settling, with subsequent correlated motion with a component perpendicular to gravity. The experimentally observed orientation distribution of the fibers exhibits a maximum, with the fiber long axis directed vertically parallel to gravity, and a smaller local maximum for particles oriented horizontally: comparison to a set of simulations by Eric Shaqfeh (see below) is particularly interesting because the secondary maximum is absent in the simulations. Also discussed were results from experiments and simulations of sedimentation of "clouds" of particles.⁴ Experiments found that an initial localized spherical cloud of particles settles with loss of a trail of particles for a long period, eventually forms a toroidal shape, and soon after

breaks up into a pair of clouds, which then may undergo further breakup. A simulation of the phenomenon using an inertialess (Stokes flow) point force description of the particles was shown to capture these phenomena, eliminating inertia as a necessary ingredient in the mechanism, and further showed the internal circulation of the particle cloud to be visually similar to the Hadamard solution for a settling drop. A final simulation result was that the initial shape of the cloud was not of great importance, as a number of shapes evolved in a similar manner toward a toroidal shape before going unstable and separating into two clouds.

Jeff Morris (City College of New York, U.S.A.) discussed mixture theory for the description of the bulk flow of suspensions, with emphasis on the role of particle-scale microstructure on the observed behavior.⁵ The microstructure of sheared concentrated suspensions was shown, based on results of Stokesian dynamics simulations, to be highly anisotropic, and this structure was related to normal stress rheology. The dynamics of extended clusters at elevated solid fraction observed in the simulations was shown to involve the formation, rotation, and disintegration of the clusters over strains of $O(1)$. Bulk mixture flow was discussed in terms of a single model approach⁶ applied to hard-sphere suspensions, Brownian suspensions, and electrorheological fluids through variation of the constitutive behavior of the stress.

Following the theme of the keynote lecture, Maria Ekiel-Jezewska (Polish Academy of Science, Poland) discussed numerical calculations of small groups of particles settling in Newtonian fluids.⁷ The simulation is based upon a singularity description of the particles in Stokes flow. Results of the simulations showed that three particles sedimenting in isolation display an interesting dynamics with two qualitatively different behaviors. In particular, there exists a critical separation of particles beginning in an equilateral configuration such that below a critical separation a periodic motion is observed, while above this separation one particle separates from the other two. Floquet theory was employed to analyze the behavior and an accompanying movie helped to visualize the phenomena.

The final three talks of the session included, among other results, novel numerical methods for describing sedimenting and sheared suspensions. In the fourth talk of the session, Eric Shaqfeh (Stanford University, U.S.A.) presented results of large-scale computational simulation of sedimentation, as well as an induced-charge electrophoresis of rod-like particles (stiff fibers).⁸ The evolution of simulation techniques applied to the problem was discussed, with recent results using a point-fiber description allowing $O(10^5)$ fibers to be simulated. The general features of the fiber orientation distribution noted in keynote talk by E. Guazzelli discussed above, in which there is a maximum with fiber axis along gravity, was found to be robust with respect to the fiber fraction and aspect ratio. However, the simulations do not capture the secondary maximum with alignment in a horizontal plane seen in experiment. Fiber motion in induced-charge electrophoresis was described, with a comparison between experiment and simulation.

In the fourth contribution of the session, Evgeny As-

molov (Central Aero-Hydrodynamic Institute, Moscow, Russia) provided a continuum description of sedimentation. This work addressed the issue noted above that vortical flows on scales large relative to the particle size are induced by fluctuations in the average density of the mixture. The density variation is due solely to concentration or particle number-density fluctuations, and as a result there arises a nonlinear coupling between the equations describing mass and momentum conservation in the system. These equations were solved in a spectral form, by expressing the fluctuations as a Fourier expansion, on a finite domain so that the front separating supernatant and sedimenting particles was resolved. The results showed vortices within the sedimenting portion, with decay in time which is similar to results observed in experiments, and a rapid fall-off of the velocity fluctuations above the front.

Eric Climent (Laboratoire de Genie Chimique, Toulouse, France) described a method developed in collaboration with Professor Martin Maxey (Brown University) termed the force-coupling method (FCM) for simulation of suspension flows, and focused his discussion on shear flow.⁹ In this method, the boundary conditions are not explicitly considered, as each particle is represented in its interaction with the suspending fluid as a single distributed singularity (e.g., as a force dipole for force-free particles) with the strength of the distribution on a Gaussian envelope. The advantages of the FCM include allowance for large particle number (systems of 4000 particles were discussed), ease of representing variable size, and inclusion of inertia. Because the method does not resolve the boundary condition precisely, consideration was restricted to concentrations up to $\phi=0.15$, with results showing agreement with other methods for the rheology of the mixture. Results for the average velocity fluctuation tensor and the related shear-induced self diffusion were also discussed, with non-Gaussian fluctuations observed at small volume fraction.

III. COMPLEX FLUIDS

The session opened with a keynote address by Octavio Manero (Universidad Nacional Autonoma de Mexico, Mexico) on “Rheology of Complex Fluids.” Manero reviewed the properties of wormlike micelles and associative polymers as observed with mechanical rheometry, rheo-optics, and rheo-NMR, including shear banding, flow curve maxima, and unexpectedly long time constants, and the use of structural kinetics models to describe the transient behavior. He then described a recent transient network model with transitions between five structural microstates that describes the shear banding and nonlinear transients such as thixotropy and stretched exponential relaxation. The model is described in Rincon *et al.*¹⁰

The session continued with three contributed papers. Todd Squires (University of California at Santa Barbara, U.S.A.) spoke on “Active and nonlinear microrheology of complex materials.” Microrheology utilizes a colloidal particle as a probe. The question investigated was the possibility of extracting nonlinear rheological information by actively driving the probe. Squires reviewed some results reported in

Squires and Brady¹¹ on the applicability of a “Generalized Stokes-Einstein Relation (GSER).” He then presented new results in which the probe is large relative to the structural component of the colloidal suspension. The bulk correction to the GSER is of the order of the colloidal particle concentration, while the bath correction is of the order of the square of the concentration.

Roger Bonnecaze (University of Texas at Austin, U.S.A.) spoke on “The flow, memory and aging of soft particle pastes.” These materials, made up of soft deformable particles packed into a dense suspension, include microgels, compressed emulsions, foams, and block copolymers. Slip is observed at stresses above the yield stress. Bonnecaze described an electrohydrodynamic lubrication theory that describes the slip and the bulk rheology at high rates (Meeker *et al.*^{12,13}). The discussion focused on a proportionality between the shear modulus and the osmotic pressure that is used in the theory.

Morton Denn (City College of New York, U.S.A.) spoke on “Unusual interfacial coupling in suspensions of liquid crystalline droplets in a flexible polymer matrix.” Dispersions of 8CB in polydimethylsiloxane form a bi-stable system that can exist either as a gel or as a dispersion of droplets (Inn and Denn¹⁴). Denn described simulated annealing calculations to determine the stable conformations in spherical and spheroidal nematic droplets, where there is a first-order transition between aligned and radial orientations, and the energy barriers between states in the bi-stable regime.

IV. FREE SURFACE FLOWS

This session included an interesting mix of free surface fluidic problems featuring drops and bubbles stabilized by surface particulates, bubbles, and wakes in viscoelastic fluids, and results concerning the deformation of drops in Newtonian fluids. The session began with a keynote talk by Howard Stone (Harvard University, U.S.A.) describing the many phenomena associated with bubbles and drops that are “armored” by particles at their interfaces.^{15,16} He began by noting that in the present technological environment, exquisite control of drop size can be obtained by creating drops in microfluidic environments. He then demonstrated a microfluidic device for creating bubbles or drops that are completely coated with 4- μm polystyrene spheres—with the coating being in a close-packed hexagonal lattice. He then made the point that any such device designed to create particle coated bubbles or drops is aided by the fact that particles at fluid-fluid interfaces always reduce the total interfacial energy of the system by an amount ΔE given by

$$\Delta E = -\gamma\pi a^2(1 + \cos \theta)^2,$$

where γ is the surface tension, a the particle radius, and θ is the contact angle with the particles. This energy reduction is typically $O(10^7)$ kT and is therefore very significant. Gas bubbles “armored” in this way are very stable to dissolution of the gas in the ambient fluid: the usual dissolution rates of 1 cm gas bubbles in water is measured in minutes, whereas armored drops are essentially stable indefinitely. Stone noted that it has been well known in the lore of ocean sailing and

fishing that very long-lived gas bubbles in the sea apparently exist because of stabilization by organic matter in the ocean, but the mechanism of this stabilization has only been the subject of speculation. He went on to present a remarkable analysis of this stabilization, first showing that the dissolution in bubbles without the particle stabilization is engendered by the capillary pressure difference between inside and outside, $\Delta P = 2\gamma/R$, where R is the particle radius with the $2/R$ factor coming from the mean curvature of the spherical bubble. This pressure ultimately drives the gas into the fluid.

Completing a series of numerical simulations of quasi-static equilibrium bubble shapes containing 122 colloidal particles on the interface, Stone demonstrated that as the gas volume inside the bubble is reduced, the surface shape of the bubble becomes one of zero mean curvature; i.e., one with no pressure difference between inside and outside. Essentially, the surface becomes one which is covered by local saddle points: thus the “armor” succeeds by supporting this rather complex shape change and thus protects the gas within. Stone finished by noting that various experiments have demonstrated that the colloidal armor actually creates an elasticity, E , in the interface which, on dimensional grounds, is $O(\gamma/a)$. This elasticity then allows for equilibrium shapes which are nonspherical and for shape multiplicity, and he showed a variety of nonspherical shapes that armored bubbles created in his laboratory could attain.

The second contribution by Pascale Aussilous (Laboratoire IUSTI, Marseille, France) demonstrated macroscopic phenomena associated with armored drops by the creation of so-called “liquid marbles.”^{17,18} In this application, a viscous liquid (usually water) is coated with hydrophobic particles. The resulting object prevents the wetting of any surface with the internal water and therefore the object acts in a manner very similar to a deformable solid. For example, as the bubble sits on a surface it adopts a shape which is nearly spherical if the radius R is smaller than the capillary length and is pancake shaped if the radius is above this length. The associated contact area with the solid surface scales differently depending on which regime of static shape exists. Dr. Aussilous demonstrated that the dynamics of the marble moving down an inclined plane for low Reynolds number can then be analyzed in terms of zero Reynolds number flow inside the marble with nearly rigid body rotation for the spherical shape, and linear shear flow defining the pancake motion. The most remarkable physical transformations occurred when the inertial forces for the marble motion were not negligible. In this case, three nonequilibrium shapes for the marble “rolling” down the inclined plane were demonstrated include the “peanut,” the “disk,” and the “wheel” shape. The wheel shape had not been demonstrated previously and was composed of a toroidal outer rim with a very thin particle covered membrane in the center. It was shown that this shape was stabilized only by contact with the solid surface so that if the marble was allowed to roll off the supporting surface, the wheel quickly collapsed into a peanut!

Following these first two talks, the session moved to the examination of cavities and bubbles in Newtonian and non-Newtonian fluids in the absence of surface stabilization. An-

drew Belmonte (Pennsylvania State University, U.S.A.) presented a study of projectile motion in wormlike micellar solutions.¹⁹ By examining the dragging of spheres and cylinders through these highly elastic solutions, he demonstrated that the wakes (in penetration through the air-fluid interface) took three forms which he described as “flow,” “cut,” and “tear.” In the flow wake, a smooth cavity is formed trailing the object. In the cut wake, the cavity broke into small bubbles, while in the tear wake, a continuous line of an air separated surface forms in the region behind the projectile. Belmonte demonstrated that the results could be collected into a phase diagram in terms of U versus d , where U is the speed of the projectile and d is the radius of the projectile. The phase boundary between cut and flow is denoted by $U/d = \text{const}$, corresponding to constant Deborah number, while the boundary between the tear and all other phases is denoted by $Ud = \text{const}$.

Patrick Anderson (Eindhoven University of Technology, The Netherlands) followed with the first of three papers on the deformation of drops in Newtonian fluids, with his work focusing on the effect of confinement on drop deformation.²⁰ Note in this context that Howard Stone, in his keynote talk, had mentioned that confinement can change qualitatively the conditions for drop breakup in microfluidic devices. Anderson demonstrated experimentally that, in Couette flow, as one decreases the distance between the parallel plates, the critical capillary number for drop breakup changes in a very different manner depending on the viscosity ratio between the inner and outer fluid, λ . For $\lambda < 1$, confinement stabilizes the drop and increases the critical capillary number, the critical conditions are unaffected by confinement for $\lambda = 1$, and for $\lambda > 1$, the drop is destabilized by confinement and the critical capillary number is reduced. Anderson followed these experimental results with boundary element simulations of a drop for $\lambda = 1$ in Couette flow using the exact Green’s function for the Stokes’ equation between parallel plates. These simulations demonstrated that even though there was in fact a strong shape change created by confinement at a given shear rate, ultimately the critical condition was unaffected.

Hector Ceniceros (University of California at Santa Barbara, U.S.A.) presented his work on the development of highly accurate boundary element simulations methods for the calculation of shape deformations in axisymmetric flows.²¹ The motivation for this work lies in understanding the high curvature states that exist in drops before coalescence. Dr. Ceniceros demonstrated that a truly fifth-order numerical method could be developed, but it required a careful subtraction of the singularities associated with the natural kernels in the integral formulation of the Stokes boundary integral method. He dubbed this method “desingularization” and showed that it was particularly important for the single-layer potential in the integral method but not the double-layer potential.

The final contribution in the session by Enrique Geffroy (Universidad Nacional Autonoma de Mexico, Mexico) involved an examination of drop deformation in so-called “mixed” flows, those planar flows that are linear combinations of pure straining motion and rotation, but with the

straining flow being dominant.^{22,23} These flows are characterized by a so-called “mixedness” parameter λ (not to be confused with the viscosity ratio defined above), which defines the eigenvectors and eigenvalues of the velocity gradient tensor. The line stretching/compression rate along these eigenvectors is proportional to $\lambda^{1/2}$, and Professor Geffroy demonstrated that these flows can be effectively generated in a two-roll mill device, where the mixedness of the flow can be controlled by the intercylinder distance (at a fixed cylinder radius) or by the radius of the cylinders (at a fixed intercylinder distance). He then showed a variety of mixed flows created in this manner along with the drop deformation engendered in these flows. Geffroy finished by presenting boundary element simulations of drops in mixed flows. The comparison with experiments and the associated critical conditions for breakup was viewed as future work. One interesting point was the the mixedness of the internal flow in the drop could be significantly larger than that of the applied external flow, and thus the breakup conditions were not linearly correlated with the line stretching rate $\lambda^{1/2}$.

V. BUBBLE-BUBBLE AND BUBBLE-WALL INTERACTIONS

The focus of this session was bubble-bubble and bubble-wall interactions. Analytical, experimental, and computational results were presented for a wide range of bubble Reynolds (Re) numbers.

In his keynote talk, Jacques Magnaudet (Institute de Mecanique des Fluides de Toulouse, France) reviewed recent theoretical, experimental, and numerical results for the interaction of a single rising bubble with a plane wall,²⁴ as well as for two rising bubbles interacting with each other.²⁵ Both low but finite Re situations and the inviscid limit were discussed. Frequently, bubbles show opposite behavior in these limits. For the example of a single rising bubble interacting with a wall, in the inviscid limit pressure forces will cause the bubble to migrate towards the wall, while for low Re the bubble is pushed away from the wall. This makes the intermediate Re range, where the crossover between these limiting cases occurs, particularly interesting to study. Experiments show that many trends are correctly predicted by asymptotic results; however, for certain parameter ranges discrepancies were observed. These point to the importance of deformation-induced migration. Furthermore, interesting novel modes of behavior were reported for $Re \sim O(100)$ or higher, such as a periodic bouncing of the bubble towards and away from the wall. Similarly, for the case of two vertically aligned, rising bubbles, the Oseen approximation predicts that the trailing bubble should catch up with the leading one, whereas in the inviscid limit the bubbles are expected to repel each other. Asymptotic analysis predicts the existence of an equilibrium separation distance at finite Re , which was confirmed by direct numerical simulations (DNS). In summary, the overview presented in this talk suggests that much understanding for intermediate Re can be gained from combining our respective knowledge of the low Re and inviscid regimes. It also points to the need of further careful experi-

ments and simulations in order to understand such novel dynamics as the periodic bouncing.

Motivated by recent observations of submicron-sized bubbles, Nicolas Bremond (Lab. Colloïdes Matériaux Divisés-ESPCI, France) reported controlled experiments involving periodic bubble arrays along a wall.²⁶ These arrays are created by heterogeneous nucleation on a patterned hydrophobic surface with micron-sized cavities. Periodic oscillations of the background pressure trigger the growth and decay of these bubble clusters. The interaction among the individual bubbles during such a cycle is analyzed both theoretically and by high-speed cinematography. When the ratio of bubble distance to bubble radius is not too small, the bubbles are weakly coupled, and their interactions can be analyzed on the basis of the Rayleigh-Plesset equation if the pressure modifications due to neighboring bubbles are taken into account. If the bubbles are located closer to each other, strong coupling occurs, and they lose their spherical shape. As they expand, they flatten and form a thin liquid film between them, which is subsequently drained by inertial forces. Eventually, the film can rupture, leading to the coalescence of the bubbles.

Ian Eames (University College London, U.K.) investigated the flow induced by bubbles as they grow on a wall, detach, move away, deform, and eventually collapse. He combined various elements from inviscid and viscous flow analysis to construct a simplified model of the flow field generated during this process, with particular emphasis to the generation of vorticity. He further discussed the relative magnitude of added mass, buoyancy, and drag forces, finding that different stages of the flows are governed by different force balances.

Dominique Legendre (Institute de Mecanique des Fluides de Toulouse, France) presented recent advances on the hydrodynamic interaction between two spherical bubbles. He extended the analysis of Magnaudet²⁵ to arbitrary angles of the distance vector with the vertical direction, and presented corresponding DNS results for moderate Reynolds numbers in the range $50 < Re < 500$. Depending on the value of Re , the bubble distance, and their orientation, the interaction can lead to the attraction or repulsion of bubbles. In addition, their center of mass can drift horizontally. The computational results point to the importance of Moore’s drag correction, and of the wake of the leading bubble. The vorticity contained in this wake can induce a significant lift force on the trailing bubble, thus resulting in a relative rotation of the bubbles, which places them side by side.

The contribution of Shu Takagi (The University of Tokyo, Japan) described several interesting observations regarding the dynamics of rising bubble clouds in the presence of surfactants. Experimental investigations²⁷ of bubble clouds in tap water typically show rapid coalescence, which quickly results in a flow dominated by fewer and larger bubbles. On the other hand, the presence of surfactants is seen to largely suppress bubble coalescence. Interestingly, different surfactant concentrations in water, as well as surfactants with very different desorption rate constants, can produce quite different dynamics. Weak surfactant concentrations result in the migration of the bubbles towards the

walls, where they form horizontal clusters. Increasing the amount of surfactant suppresses this migration, which points to the importance of Marangoni effects with regard to the lift force in shear flows. This topic was addressed by numerical simulations.²⁸ As a first step, a stagnant cap model is considered, with an area free of tangential stress near the front stagnation point and a no-slip surface near the rear. The lift force is seen to decrease as the no-slip surface area near the rear of the bubble increases. As a next step, the author solves the balance equation for the surfactant concentration on the bubble surface in order to obtain a more detailed model. He finds that for surfactants with high desorption rates, the bubble surface remains nearly clean, so that Marangoni stresses are weak. For small desorption rates, in contrast, high surface concentrations are obtained at the rear of the bubble. These result in large tangential stresses, similar to the stagnant cap model, and in a reduction of the lift force.

Abraham Medina (Instituto Mexicano del Petroleo, Mexico) presented a theoretical and experimental investigation of the growth and detachment of bubbles at submerged orifices.²⁹ By varying the orifice material, the influence of the contact angle is analyzed, along with that of the orifice radius and the volumetric flow rate. Experimental data are compared with simulation results based on the Laplace equation for the velocity potential, and good agreement is obtained. Normalization with the Fritz volume and the critical flow rate is seen to collapse the data.

VI. GRANULAR FLOWS WITH INTERSTITIAL FLUID EFFECTS

This session was devoted to granular materials, but in many cases with an interstitial liquid present. This included the keynote of the session, in which Melany Hunt (California Institute of Technology, U.S.A.) described collisional particle interactions in liquid-solid suspensions and their effect on the mixture rheology. The work was motivated by well-known work of Bagnold in which scaling of stresses in shear flow of a concentrated neutrally buoyant liquid-solid mixture were argued to transition from linear to quadratic dependence on shear rate as shear rate increased. A crucial issue discussed and followed by questions and discussion is that of the inertial scaling of stress (as argued by Bagnold³⁰). Experiments designed to develop an understanding and simple model of the collision of a pair of particles in liquid were described.³¹ In these, a crucial input parameter is the Stokes number $St = mU/6\pi\mu a^2$. For the collision with a wall, the effective coefficient of restitution (ϵ_{eff}) was found to increase with St above a critical St of about 10, below which there is zero rebound and hence $\epsilon_{\text{eff}}=0$. A similar behavior was seen for collision of a pair of particles, as below a critical St in this problem the pair does not show any rebound, but instead collides and moves together.³² For an oblique angle of impact, the roughness and frictional interaction of the particle with a wall were shown to play a key role, which results in significant variations between particles of different materials. A discussion of the detailed issues in the near-contact region, including asperity contact, elastohydrodynamics, and the interactions of these two were discussed.

Eckart Meiburg (University of California at Santa Barbara, U.S.A.) discussed the two-way coupling between particles and turbulence and its impact upon the sedimentation/settling rate.³³ The system studied was dense particles in gas flow at solids fractions of $O(10^{-3})$, such that mean separations of particles were large relative to their size, but the mass loading fraction (solid to gas) can be comparable to unity. Simulations were compared to previous experiments, themselves very disparate in their results, in an effort to resolve the basis for enhanced settling rate in turbulent flow. The simulations employed a Fourier pseudospectral method and Lagrangian particle tracking, with the underlying fluid motion a turbulence forced by the method proposed by Eswaran and Pope.³⁴ A goal was to distinguish between one- and two-way coupling, with the one-way coupling validated against prior approaches. Both one- and two-way coupling were found to induce large-scale heterogeneity of the solid loading and hence to enhance settling; this effect was found to grow weaker as the loading increased for the two-way coupling. A discussion of the possible basis for discrepancies with the experiments was presented.

Osamu Sano (Tokyo University of Agriculture and Technology, Japan) described an experimental and numerical study of the collapse of cavities in a porous medium or granular flow.³⁵ The experiments considered were prepared by laying down a layer of granular material on a plate with one or two cylinders placed in the layer to maintain a cavity or cavities, followed by removal of the cylinder(s) and sandwiching the layer by a second glass plate. A pressure-driven flow was imposed through the material. The critical velocity for onset of cavity collapse, when viscous stress overcomes the interparticle friction and the confining pressure, was determined; when two cavities were present at separations comparable to their size, there was significant interaction owing to the distortion of flow field (streamlines) toward the high-permeability (vanishing Darcy resistance) cavities, and at fixed separation there is dependence on the angle of the pair relative to the flow. Interestingly, it was shown that the downstream cavity in a pair collapses first, a result which may be explained by the fact, shown by simulation, that streamlines are drawn toward the cavities, and the second cavity thus sees a greater flow rate. The collapse of cavities was reported in quantitative terms through the rate of area collapse in this quasi-two-dimensional system, and the manner in which the collapse propagates, as well as fluidization of the porous packs, was demonstrated by video.

Francois Charru (Institute de Mecanique des Fluides de Toulouse, France) discussed wavelength selection in the formation of rippled patterns in a settled granular bed beneath a shear flow.³⁶ The behavior was discussed in terms of a critical shear stress, τ_c , rather than the dimensionless Shields parameter of $\tau/\Delta\rho gd$ (d is the particle size and the density difference is between particle and liquid). Above but near τ_c , settled beds of particles over a range of different sizes show a rippling that evolves over time, typically coarsening in structure; for $\tau \gg \tau_c$, the evolution is more rapid and simpler. Continuum modeling of the bed evolution considering a shear flow over a sinusoidal bedform with resuspension mechanisms incorporated was discussed. It was shown that a

flux of particles out of phase with the bedform leads to formation of crests on the settled bed. In a comparison to experiments, it was found that it is very difficult to obtain agreement with the observed wavelength for turbulent shear flow, and while the scaling for ripples under a viscous shear flow agrees with experiment, the predictions differ significantly in magnitude. Erosion-deposition modeling including bedform friction was also discussed.

Lou Kondic (New Jersey Institute of Technology, U.S.A.) described simulation of a sheared granular material subject to both steady and variable gravity.³⁷ The configuration considered was a Couette geometry with its cylinder axis aligned with gravity. The simulation used an event-driven method with the particles described by a soft interaction potential. The results showed rheology that varied significantly with the driving rate relative to particle weight. For small stresses, a yield stress was observed, and with increasing stress this gave way to first a linear and then a quadratic shear rate scaling of the stresses. The role of vibrations (imposed along the gravity direction) in combination with shearing in heavy particle systems was discussed as well.

Philippe Gondret (Laboratoire FAST, Orsay, France) described the behavior of avalanches both under liquid and in air.³⁸ The avalanches were generated experimentally by rotating a short drum containing sand-scale particles in either air or a liquid; results of experiments in which a flow was imposed past a settled bed in a Hele-Shaw channel of various inclinations were also discussed. Avalanches of approximately three particle diameters in thickness were observed. Results on the duration of the avalanches, the evolution of the surface angle, and the grain flux were presented. Modeling of the phenomena was presented by a method that followed Cassar *et al.*³⁹ in which a key dimensionless parameter is the product of the shear rate and the fall time of a particle in the fluid.

VII. BUBBLES AND DROPS IN NON-NEWTONIAN MEDIA

The session opened with a keynote address by Roberto Zenit (Universidad Nacional Autonoma de Mexico, Mexico) on “The motion of an air bubble in a non-Newtonian liquid: Conditions for the appearance of the bubble velocity discontinuity.” Astarita and Apuzzo⁴⁰ first observed that bubbles rising in aqueous polymer solutions experience a discontinuity in velocity (with respect to bubble volume) that is accompanied by the formation of a cusp at the trailing end. Zenit reported on bubble rise experiments in aqueous and non-aqueous non-Newtonian solutions with both shear-thinning and relatively constant viscosities and with and without significant normal stresses.^{41,42} He and his co-workers identified the shape transition and velocity discontinuity with an elastic effect that correlated with values of the dimensionless group $N_1 d / \sigma$ greater than unity, where N_1 is the first normal stress difference measured in shear, d is the diameter, and σ is the interfacial tension. The discussion focused on the use of an elastic measure obtained in shear flow rather in extension, on the estimate of the effective shear rate for evaluating the elastic stress, and on the possible effect of surfactants. The

session continued with four contributed papers.

Denis Rodrigue (Université Laval, Canada) described the effect of varying interfacial tension on drop motion by surfactant addition when one phase is an aqueous polymer solution and one is a Newtonian fluid. The droplet shape is affected by the surfactant concentration, whether the continuous phase is the Newtonian or polymeric fluid, and whether the motion is up or down, but terminal velocities were relatively insensitive to surfactant concentration when a suitably renormalized modified Eötvös number is used. The eccentricity data collapse on a universal curve when plotted versus this modified Eötvös number.

Carlos Málaga (Universidad Nacional Autonoma de Mexico, Mexico) reported on the use of a boundary integral method to study the shape of a bubble rising in a dilute polymer solution using a finitely extensible nonlinear elastic model.⁴³ The analysis ignores the wake and evaluates the volume integral of the stress divergence only in selected regions. High curvature was found near the rear stagnation region of the bubble, and curvature was found to diverge at a critical Bond number, although the discontinuity in rise velocity was not found.

Javier Diez (Universidad Nacional del Centro de la Provincia de Buenos Aires, Argentina) described the “Pearling process on a fluid strip on a partially wetting surface,” wherein a long strip of viscous fluid on a horizontal glass substrate under partial wetting conditions breaks up into droplets through a process of successive bulge growth and pinchoff from the two ends.⁴⁴ The pinching process forms a chain of satellite drops. All droplets have a nearly elliptical shape (aspect ratio approximately 1.25) resulting from non-uniform contact angle along the perimeter. A model based on an energy balance with dissipation proportional to the area covered by the droplets predicts the number of droplets.

Raúl Montiel (Universidad Autonoma Metropolitana, Mexico) discussed “Some aspects of the deformation of flexible particles in a mobile interphase.”⁴⁵ Elastic rubber particles were placed near an advancing liquid front in a tube (“fountain flow”). The particles protrude from the interface and deform. The height of the bump was determined from an equilibrium between capillary and drag forces for a given particle deformation. The data were analyzed using an extension of a treatment for rigid particles in the same flow by Hoffman.⁴⁶

VIII. MULTIPHASE MICROFLUIDICS

The final session concerned systems in which fluid/fluid interfaces play a key role with particular emphasis on microfluidics and small scale systems.

Detlef Lohse (University of Twente, The Netherlands) began the session with an invited lecture on the dynamics of small bubbles. He framed his discussion by discussing single-bubble sonoluminescence (SBSL),⁴⁷ and the variety of physical phenomena required for its understanding: fluid mechanics via the Reyleigh-Plesset equation, shape and chemical stability, gaseous diffusion and dissolution, and energy focusing. He dubbed the phenomenon the “hydrogen atom” of bubble dynamics, in that understanding SBSL gives the

intuition required to build up more complicated systems and behaviors in bubble dynamics—multiple bubbles, interactions between bubbles and walls, and so on. He then moved on to a curious problem in inkjet printing,⁴⁸ which might otherwise seem unrelated, in which perfectly reproducible ink drops are created using a piezoelectric actuator for billions of cycles, and then suddenly fail. To study this failure, Lohse's group used the piezo-actuator to "listen" to the droplet formation process, and would "hear" the failure well in advance of seeing it. Further study revealed that bubbles (introduced when a dust particle hits the free ink surface, or deliberately with a pool of ink) are to blame. He closed by remarking that they would not have understood this phenomenon at all without the knowledge gained from a purely academic study on SBSL.

Eduardo Ramos (Universidad Nacional Autonoma de Mexico, Mexico) presented work designed to make efficient heat exchangers by boiling—where the bubbles then carry significant (latent) heat per unit mass. He described experiments designed to nucleate the bubbles in prescribed ways—e.g., by introducing cavities into the heating element—then measured and modeled the time series of the bubbles that were produced, and identified mechanisms responsible.⁴⁹

Wendy Zhang (University of Chicago, U.S.A.) discussed systems in which fluid/fluid free surfaces undergo topological transitions. She discussed so-called selective withdrawal in which two fluid layers sit atop each other, and fluid is drawn into a tube in the upper fluid.⁵⁰ Below a critical flow rate, a stable hump appears, whereas above the critical flow rate a transition occurs to form a spout. She concluded that the dynamic length scales (spout height and tip curvature) are set by the smaller of two "external" length scales—either the distance to the spout, or the capillary length. Finally, she found that no hysteresis appears in the transition.

Shelley Anna (Carnegie Mellon University, U.S.A.) presented studies in a microfluidic flow focusing device that were motivated by the observation that droplets that one can create in such microdevices is generally limited by the feature size of the device. Her objectives were to explore ways in which dramatically smaller droplets could be generated.⁵¹ She discussed work on droplet formation with "clean" interfaces, and moved on to discussing studies with fluids containing soluble surfactant. In the presence of such surfactants, she observed that tip streaming could be driven, during which process significantly smaller droplets are formed. She explored parameter space and found that tip streaming occurs in the regime where one would expect it to from classic theory.

Dominique Langevin (Université Paris-Sud, France) discussed fluid drainage through foams, which she presented as analogous to flow through a porous medium with variable porosity; whether the flow through each "pore" behaves more like plug flow or Poiseuille flow depends on whether the retarding stresses come from the viscous stresses in the bulk, or from surface viscosity (i.e., the Lemlich parameter). A variety of experiments⁵² demonstrated the transition between these two regimes. She then described experiments with non-Newtonian fluids. Forced-drainage experiments show that the behavior of Newtonian solutions and of shear-

thinning ones (foaming solutions containing either Carbopol or Xanthan) are identical, provided one considers the actual viscosity corresponding to the shear rate found inside the foam. She also showed that foams made of viscoelastic solutions counterintuitively drain faster than foams made with Newtonian solutions of the same viscosity.

Finally, Sascha Hilgenfeldt (Northwestern University, U.S.A.) closed the session and the Symposium by talking about fracturing foams. He described experiments in which a fluid was driven into two-dimensional foam within a Hele-Shaw cell.⁵³ At low intrusion rates, a Saffman-Taylor-like finger formed (although it did not fill half the cell, as in the standard viscous case). At high intrusion rates, a rapid crack was observed to penetrate the cell. He drew an analogy between these two modes of finger propagation and ductile and brittle fracture in metals.

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¹See EPAPS Document No. E-PHFLE6-18-030611 for a book of abstracts from the conference. This document can be reached via a direct link in the online article's HTML reference section or via the EPAPS homepage (<http://www.aip.org/pubservs/epaps.html>).

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