12.8

12.8 A centrifugal water pump having an impeller diameter of 0.5 m operates at 900 rpm. The water enters the pump parallel to the pump shaft. If the exit blade angle, $\beta_2$, (see Fig. 12.8) is 25°, determine the shaft power required to turn the impeller when the flow through the pump is 0.16 $m^3/s$. The uniform blade height is 50 mm.

$$\dot{W}_{shaft} = T_{shaft}\,\omega = T_{shaft}\,2\pi N$$

$$T_{shaft} = \rho Q\,(r_2 V_{\theta 2} - r_1 V_{\theta 1}) \qquad (\text{Eq. } 12.10)$$

With $V_{\theta 1} = 0$

$$T_{shaft} = \rho Q\, r_2 V_{\theta 2} \qquad (1)$$

From Fig. 12.8c

$$\cot\beta_2 = \frac{U_2 - V_{\theta 2}}{V_{r2}}$$

so that

$$V_{\theta 2} = U_2 - V_{r2}\cot\beta_2 \qquad (2)$$

For $r_2 = \frac{0.5\,m}{2} = 0.25\,m$ with $\omega = \frac{\left(900\,\frac{rev}{min}\right)\left(2\pi\,\frac{rad}{rev}\right)}{60\,\frac{s}{min}} = 94.2\,\frac{rad}{s}$

then

$$U_2 = r_2\omega = (0.25\,m)\left(94.2\,\frac{rad}{s}\right) = 23.6\,\frac{m}{s}$$

Since the flowrate is given, it follows that

$$Q = 2\pi r_2 b_2 V_{r2}$$

or

$$V_{r2} = \frac{Q}{2\pi r_2 b_2} = \frac{\left(0.16\,\frac{m^3}{s}\right)}{(2\pi)(0.25\,m)(0.05\,m)} = 2.04\,\frac{m}{s}$$

Thus, from Eq. (2)

$$V_{\theta 2} = (23.6 - 2.04\cot 25^\circ)\,\frac{m}{s} = 19.2\,\frac{m}{s}$$

and from Eq. (1)

$$T_{shaft} = \left(999\,\frac{kg}{m^3}\right)\left(0.16\,\frac{m^3}{s}\right)(0.25\,m)\left(19.2\,\frac{m}{s}\right) = 768\,N\cdot m$$

so,

$$\dot{W}_{shaft} = (768\,N\cdot m)\left(2\pi\,\frac{rad}{rev}\right)\left(900\,\frac{rev}{min}\right)\frac{1}{\left(60\,\frac{s}{min}\right)}\left(\frac{1}{1000\,\frac{N\cdot m}{s\cdot kW}}\right) \cancel{= 0.08\,kW}\ \ 72.4\,kW$$

12.15

**12.15** The performance characteristics of a certain centrifugal pump having a 9-in.-diameter impeller and operating at 1750 rpm are determined using an experimental setup similar to that shown in Fig. 12.10. The following data were obtained during a series of tests in which $z_2 - z_1 = 0$, $V_2 = V_1$, and the fluid was water.

| Q (gpm) | 20 | 40 | 60 | 80 | 100 | 120 | 140 |
|---|---|---|---|---|---|---|---|
| $p_2 - p_1$ (psi) | 40.2 | 40.1 | 38.1 | 36.2 | 33.5 | 30.1 | 25.8 |
| Power input (hp) | 1.58 | 2.27 | 2.67 | 2.95 | 3.19 | 3.49 | 4.00 |

Based on these data, show or plot how the actual head rise, $h_a$, and the pump efficiency, $\eta$, vary with the flowrate. What is the design flowrate for this pump?

From Eq. 12.19 with $z_1 = z_2$ and $V_1 = V_2$

$$h_a = \frac{p_2 - p_1}{\gamma}$$

Thus, for the first set of data in the table

$$h_a = \frac{\left(40.2 \frac{lb}{in.^2}\right)\left(144 \frac{in.^2}{ft^2}\right)}{62.4 \frac{lb}{ft^3}} = 92.8 ft$$

From Eq. 12.23

$$\eta = \frac{\gamma Q h_a / 550}{bhp}$$

and for the first set of data in the table

$$\eta = \frac{\left(62.4 \frac{lb}{ft^3}\right)\left[(20\, gpm)/\left(7.48 \frac{gal}{ft^3}\right)\left(60 \frac{s}{min}\right)\right](92.8 ft)}{(1.58\, hp)\left(550 \frac{ft\cdot lb}{s\cdot hp}\right)}$$

$$= 0.297$$

or

$$\eta = 29.7\%$$

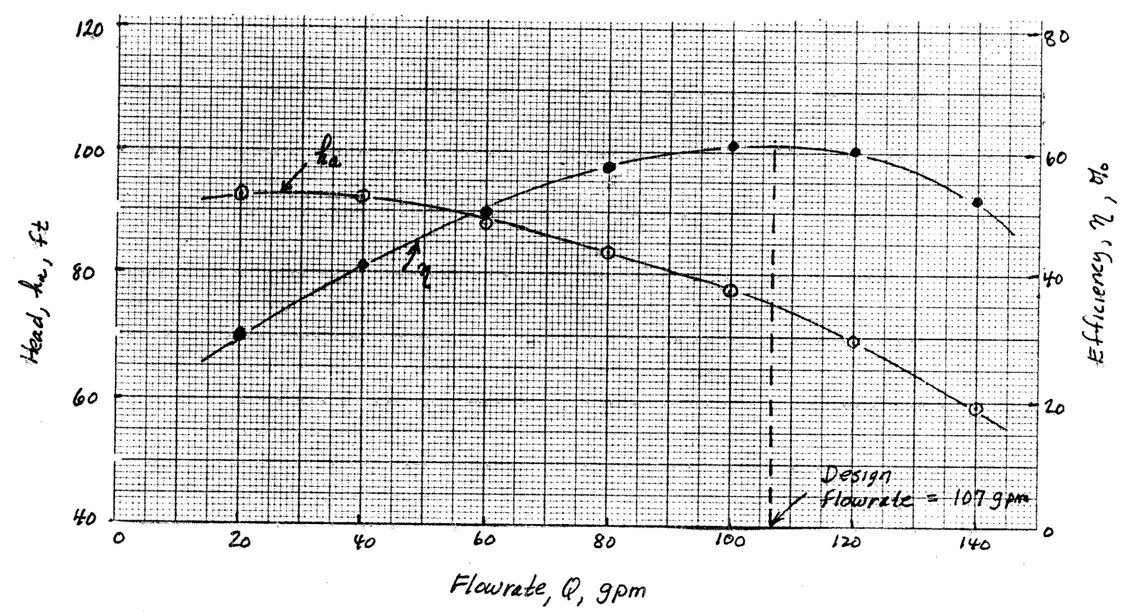
Remaining values for $h_a$ and $\eta$ can be calculated in a similar manner, and all values are tabulated in the table below.

| Q (gpm) | 20 | 40 | 60 | 80 | 100 | 120 | 140 |
|---|---|---|---|---|---|---|---|
| $h_a$ (ft) | 92.8 | 92.5 | 87.9 | 83.5 | 77.3 | 69.5 | 59.5 |
| $\eta$ (%) | 29.7 | 41.2 | 49.9 | 57.5 | 61.3 | 60.4 | 52.6 |

(con't)

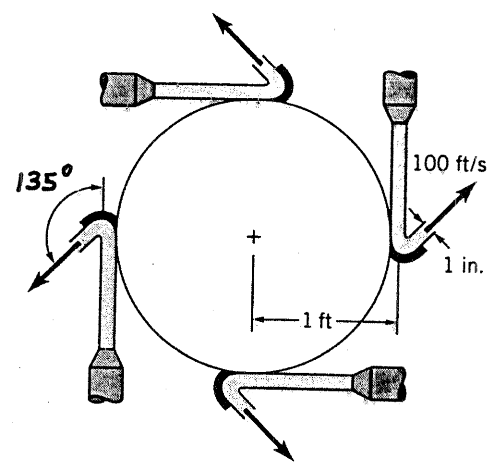
12.15 (con't)

A plot of the data is shown below. The design flowrate occurs at peak efficiency and is 107 gpm.



12.41

12.41 A Pelton wheel turbine is illustrated in Fig. P12.41. The radius to the line of action of the tangential reaction force on each vane is 1 ft. Each vane deflects fluid by an angle of 135° as indicated. Assume all of the flow occurs in a horizontal plane. Each of the four jets shown strikes a vane with a velocity of 100 ft/s and a stream diameter of 1 in. The magnitude of velocity of the jet remains constant along the vane surface. **(a)** How much torque is required to hold the wheel stationary? **(b)** How fast will the wheel rotate if shaft torque is negligible and what practical situation is simulated by this condition?




■ FIGURE P12.41

$$T = n\dot{m} r_m (U - V_1)(1 - \cos\beta) \quad \text{where } n = 4 \tag{1}$$

(a) With the wheel stationary $U = 0$ so that

$T = -4\dot{m} r_m V_1 (1 - \cos\beta)$ where

$$\dot{m} = \rho A V = \left(1.94 \frac{slugs}{ft^3}\right)\frac{\pi}{4}\left(\frac{1}{12} ft\right)^2 \left(100 \frac{ft}{s}\right) = 1.057 \frac{slugs}{s}$$

Thus, $T = -4\left(1.057 \frac{slugs}{s}\right)(1 ft)\left(100 \frac{ft}{s}\right)(1 - \cos 135°) = \underline{\underline{-722 \ ft \cdot lb}}$

(b) From Eq. (1), when $T = 0$, then $U = V_1$

Thus,

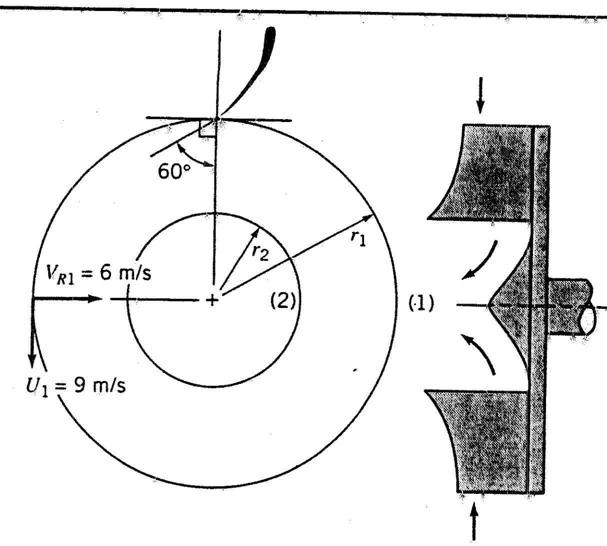
$$U = \omega r_m = V_1 \quad \text{or} \quad \omega = \frac{V_1}{r_m} = \frac{100 \frac{ft}{s}}{1 ft} = 100 \frac{rad}{s}\left(\frac{60 s}{min}\right)\left(\frac{1 rev}{2\pi \ rad}\right)$$

$$= \underline{\underline{955 \ rpm}}$$

The zero torque case represents a broken shaft situation.

12.46

12.46 An inward flow radial turbine (see Fig. P12.46) involves a nozzle angle, $\alpha_1$, of 60° and an inlet rotor tip speed, $U_1$, of 9 m/s. The ratio of rotor inlet to outlet diameters is 2.0. The radial component of velocity remains constant at 6 m/s through the rotor and the flow leaving the rotor at section (2) is without angular momentum. **(a)** If the flowing fluid is water and the stagnation pressure drop across the rotor is 110 kPa, determine the loss of available energy across the rotor and the efficiency involved. **(b)** If the flowing fluid is air and the static pressure drop across the rotor is 0.07 kPa, determine the loss of available energy across the rotor and the rotor efficiency.



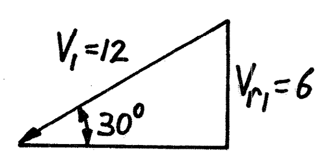

■ FIGURE P12.46

(a) $\text{loss} = \dfrac{p_{01}-p_{02}}{\rho} + w_{shaft}$, where $p_{01}-p_{02}$ = stagnation pressure drop across rotor $= \Delta p_s$

and

$w_{shaft} = U_2 V_{\theta 2} - U_1 V_{\theta 1} = -U_1 V_{\theta 1}$ since $V_{\theta 2} = 0$

Thus, $w_{shaft} = -(9\,\frac{m}{s})(12\,\frac{m}{s}\cos 30°) = -93.5\,\frac{m^2}{s^2}$




$V_{\theta 1} = V_1 \cos 30°$

so that

$$\text{loss} = \frac{110\times 10^3\,\frac{N}{m^2}}{999\,\frac{kg}{m^3}} + \left(-93.5\,\frac{m^2}{s^2}\right) = \underline{\underline{16.6\,\frac{m^2}{s^2}}}$$

Also,

$$\eta = \frac{-w_{shaft}}{\frac{\Delta p_s}{\rho}} = \frac{93.5\,\frac{m^2}{s^2}}{\frac{(110\times 10^3\,\frac{N}{m^2})}{(999\,\frac{kg}{m^3})}} = \underline{\underline{0.849}}$$

(con't)

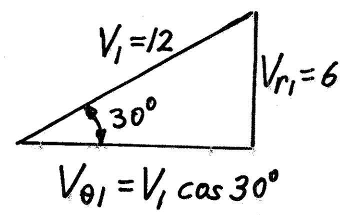
12.46 (con't)

(b) $\text{loss} = \dfrac{p_{01}-p_{02}}{\rho} + w_{shaft}$ , where $p_{01}-p_{02}$ = stagnation pressure drop across rotor = $\Delta p_s$

and

$$w_{shaft} = U_2 V_{\theta 2} - U_1 V_{\theta 1} = -U_1 V_{\theta 1} \quad \text{since } V_{\theta 2}=0$$

Thus, $w_{shaft} = -(9\,\frac{m}{s})(12\,\frac{m}{s}\cos 30^\circ) = -93.5\,\frac{m^2}{s^2}$




Also,

$$\Delta p_s = p_1 - p_2 + \tfrac{1}{2}\rho(V_1^2 - V_2^2)$$

$$= 0.07\text{ kPa} + \tfrac{1}{2}(1.23\,\tfrac{kg}{m^3})((12\,\tfrac{m}{s})^2 - (6\,\tfrac{m}{s})^2)\left(\frac{1\text{ kPa}}{10^3\text{ Pa}}\right)$$

$$= (0.07 + 0.0664)\text{ kPa} = 0.1364\text{ kPa}$$

Thus,

$$\text{loss} = \frac{0.1364\times 10^3\,\frac{N}{m^2}}{(1.23\,\frac{kg}{m^3})} - 93.5 = \underline{\underline{17.4\,\frac{m^2}{s^2}}}$$

and

$$\eta = \frac{-w_{shaft}}{\left(\frac{\Delta p_s}{\rho}\right)} = \frac{93.5\,\frac{m^2}{s^2}}{\left(\dfrac{.1364\,\frac{N}{m^2}}{1.23\,\frac{kg}{m^3}}\right)} = \underline{\underline{0.843}}$$