5.68

**5.68** Water enters a rotating lawn sprinkler through its base at the steady rate of 16 gal/min as shown in Fig. P5.68. The exit cross section area of each of the two nozzles is 0.04 in.$^2$ and the flow leaving each nozzle is tangential. The radius from the axis of rotation to the centerline of each nozzle is 8 in. **(a)** Determine the resisting torque required to hold the sprinkler head stationary. **(b)** Determine the resisting torque associated with the sprinkler rotating with a constant speed of 500 rev/min. **(c)** Determine the angular velocity of the sprinkler if no resisting torque is applied.

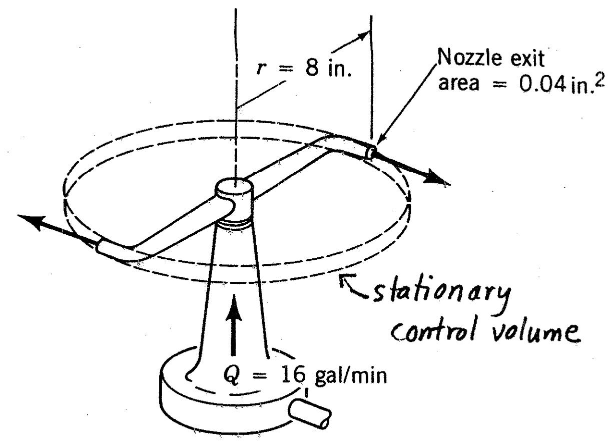



**FIGURE P5.68**

This is similar to Example 5.17.

(a) To determine the resisting torque required to hold the sprinkler head stationary we use the moment-of-momentum torque equation (Eq. 5.50). Thus,

$$T_{shaft} = \dot{m} r_2 V_{\theta 2} = \rho Q r_2 V_{\theta,2} \qquad (1)$$

For $V_{\theta,2}$ we use

$$V_{\theta 2} = \frac{Q}{2A_{\substack{nozzle\\exit}}} = \frac{\left(16\,\frac{gal}{min}\right)\left(144\,\frac{in.^2}{ft^2}\right)}{2\,(0.04\,in.^2)\left(7.48\,\frac{gal}{ft^3}\right)\left(60\,\frac{s}{min}\right)}$$

or

$$V_{\theta,2} = 64.17\,\frac{ft}{s}$$

With Eq. 1 we obtain

$$T_{shaft} = \frac{\left(1.94\,\frac{slugs}{ft^3}\right)\left(16\,\frac{gal}{min}\right)(8\,in.)\left(64.17\,\frac{ft}{s}\right)\left(1\,\frac{lb}{slug\cdot\frac{ft}{s^2}}\right)}{\left(7.48\,\frac{gal}{ft^3}\right)\left(60\,\frac{s}{min}\right)\left(12\,\frac{in.}{ft}\right)}$$

and

$$T_{shaft} = \underline{\underline{2.96\ ft\cdot lb}}$$

(b) To determine the resisting torque associated with a sprinkler speed of 500 $\frac{rev}{min}$ we use Eq. 1 again. However, with rotation we have

$$V_{\theta,2} = W_2 - U_2 \qquad (2)$$

For $W_2$ we use

$$W_2 = \frac{Q}{2A_{\substack{nozzle\\exit}}} = \frac{\left(16\,\frac{gal}{min}\right)\left(144\,\frac{in.^2}{ft^2}\right)}{(2)(0.04\,in.^2)\left(7.48\,\frac{gal}{ft^3}\right)\left(60\,\frac{s}{min}\right)} = 64.17\,\frac{ft}{s}$$

(con't)

5.68 (con't)

For $U_2$ we use

$$U_2 = r_2\omega = \frac{(8\text{ in.})\left(500\frac{\text{rev}}{\text{min}}\right)\left(2\pi\frac{\text{rad}}{\text{rev}}\right)}{\left(12\frac{\text{in.}}{\text{ft}}\right)\left(60\frac{\text{s}}{\text{min}}\right)} = 34.91\frac{\text{ft}}{\text{s}}$$

Thus with Eq. 2 we have

$$V_{\theta,2} = 64.17\frac{\text{ft}}{\text{s}} - 34.91\frac{\text{ft}}{\text{s}} = 29.26\frac{\text{ft}}{\text{s}}$$

and with Eq. 1 we obtain

$$T_{\text{shaft}} = \frac{\left(1.94\frac{\text{slugs}}{\text{ft}^3}\right)\left(16\frac{\text{gal}}{\text{min}}\right)(8\text{ in.})\left(29.26\frac{\text{ft}}{\text{s}}\right)}{\left(7.48\frac{\text{gal}}{\text{ft}^3}\right)\left(60\frac{\text{s}}{\text{min}}\right)\left(12\frac{\text{in.}}{\text{ft}}\right)}\left(1\frac{\text{lb}}{\text{slug}\cdot\frac{\text{ft}}{\text{s}^2}}\right)$$

and

$$T_{\text{shaft}} = \underline{\underline{1.35\text{ ft}\cdot\text{lb}}}$$

(c) To determine the angular velocity of the sprinkler if no resisting torque is applied we use the combination of Eqs. 1 and 2 to obtain

$$U_2 = W_2$$

or

$$\omega = \frac{W_2}{r_2} = \frac{\left(64.17\frac{\text{ft}}{\text{s}}\right)\left(12\frac{\text{in.}}{\text{ft}}\right)}{(8\text{ in.})} = 96.3\frac{\text{rad}}{\text{s}}$$

The rotor speed, $N$, is thus

$$N = \left(96.3\frac{\text{rad}}{\text{s}}\right)\frac{\left(60\frac{\text{s}}{\text{min}}\right)}{\left(2\pi\frac{\text{rad}}{\text{rev}}\right)} = \underline{\underline{920\frac{\text{rev}}{\text{min}}}}$$

5.74

**5.74** A fan (see Fig. P5.74) has a bladed rotor of 12-in.-outside diameter and 5-in.-inside diameter and runs at 1725 rpm. The width of each rotor blade is 1 in. from blade inlet to outlet. The volume flowrate is steady at 230 ft³/min and the absolute velocity of the air at blade inlet, $V_1$, is purely radial. The blade discharge angle is 30° measured with respect to the tangential direction at the outside diameter of the rotor. **(a)** What would be a reasonable blade inlet angle (measured with respect to the tangential direction at the inside diameter of the rotor)? **(b)** Find the power required to run the fan.

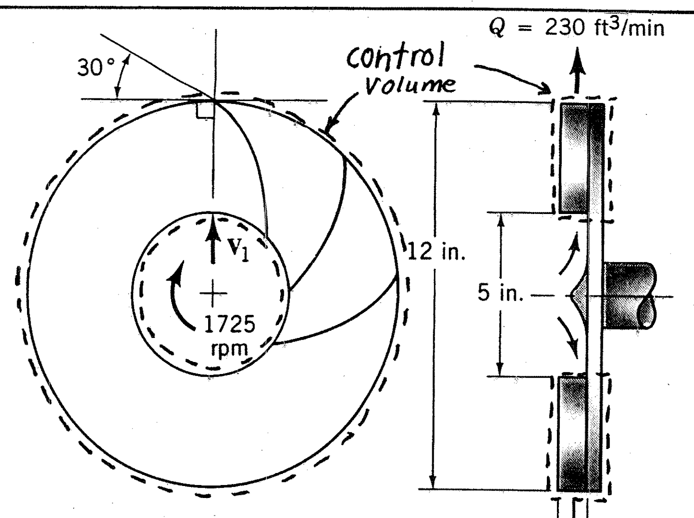
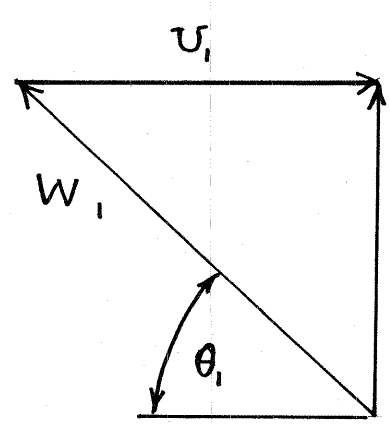



**FIGURE P5.74**

*The stationary and non-deforming control volume shown in the sketch above is used. To determine a reasonable blade inlet angle we assume that the blade should be tangent to the relative velocity at the inlet. The inlet velocity triangle is sketched below.*




*With the velocity triangle, we conclude that*

$$\theta_1 = \tan^{-1}\left(\frac{V_1}{U_1}\right) \qquad (1)$$

*Now*

$$V_1 = \frac{Q}{A_1} = \frac{Q}{2\pi r_1 h_1} = \frac{\left(230\,\frac{ft^3}{min}\right)\left(144\,\frac{in.^2}{ft^2}\right)}{2\pi\,(2.5\text{ in.})(1\text{ in.})\left(60\,\frac{s}{min}\right)} = 35.1\,\frac{ft}{s}$$

*and*

$$U_1 = r_1\omega = \frac{(2.5\text{ in.})}{\left(12\,\frac{in.}{ft}\right)}\frac{\left(1725\,\frac{rev}{min}\right)\left(2\pi\,\frac{rad}{rev}\right)}{\left(60\,\frac{s}{min}\right)} = 37.6\,\frac{ft}{s}$$

*Thus with Eq. 1*

$$\theta_1 = \tan^{-1}\left[\frac{\left(35.1\,\frac{ft}{s}\right)}{\left(37.6\,\frac{ft}{s}\right)}\right] = \underline{\underline{43^\circ}}$$

(con't)

5.74 (con't)

The power required, $\dot{W}_{shaft}$, may be obtained with Eq. 5.53. Thus

$$\dot{W}_{shaft} = \dot{m}_2 U_2 V_{\theta,2} \qquad (2)$$

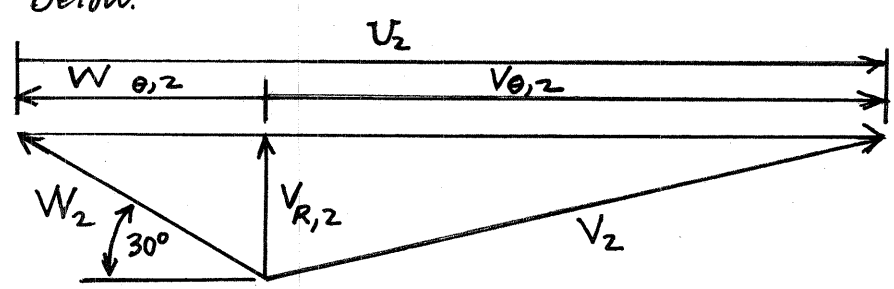
The mass flowrate, $\dot{m}_2$, may be obtained as follows.

$$\dot{m}_2 = \rho Q = \left(2.38 \times 10^{-3} \frac{slugs}{ft^3}\right)\left(230 \frac{ft^3}{min}\right)\left(\frac{1}{60 \frac{s}{min}}\right) = 9.12 \times 10^{-3} \frac{slug}{s}$$

Also

$$U_2 = r_2 \omega = \frac{(6 \text{ in.})\left(1725 \frac{rev}{min}\right)\left(2\pi \frac{rad}{rev}\right)}{\left(12 \frac{in.}{ft}\right)\left(60 \frac{s}{min}\right)} = 90.3 \frac{ft}{s}$$

The value of $V_{\theta,2}$ may be obtained by considering the velocity triangle for the flow leaving the rotor at section (2). The relative velocity at the rotor exit is considered to be tangent to the blade there. The rotor exit flow velocity triangle is sketched below.




Now

$$V_{\theta,2} = U_2 - W_{\theta,2}$$

and

$$W_{\theta,2} = \frac{V_{R,2}}{\tan 30^\circ} = \frac{\frac{Q}{2\pi r_2 h_2}}{\tan 30^\circ} = \frac{\dfrac{\left(230 \frac{ft^3}{min}\right)\left(144 \frac{in.^2}{ft^2}\right)}{2\pi (6 \text{ in.})(1 \text{ in.})\left(60 \frac{s}{min}\right)}}{\tan 30^\circ} = 25.4 \frac{ft}{s}$$

Thus

$$V_{\theta,2} = 90.3 \frac{ft}{s} - 25.4 \frac{ft}{s} = 64.9 \frac{ft}{s}$$

and from Eq. 2

$$\dot{W}_{shaft} = \left(9.12 \times 10^{-3} \frac{slug}{s}\right)\left(90.3 \frac{ft}{s}\right)\left(64.9 \frac{ft}{s}\right)\left(\frac{1 \text{ lb}}{slug \cdot \frac{ft}{s^2}}\right) = \underline{\underline{53.4}} \frac{ft \cdot lb}{s}$$

5.76

**5.76** A sketch of the arithmetic mean radius blade sections of an axial-flow water turbine stage is shown in Fig. P5.76. The rotor speed is 1000 rpm. **(a)** Sketch and label velocity triangles for the flow entering and leaving the rotor row. Use **V** for absolute velocity, **W** for relative velocity, and **U** for blade velocity. Assume flow enters and leaves each blade row at the blade angles shown. **(b)** Calculate the work per unit mass delivered at the shaft.

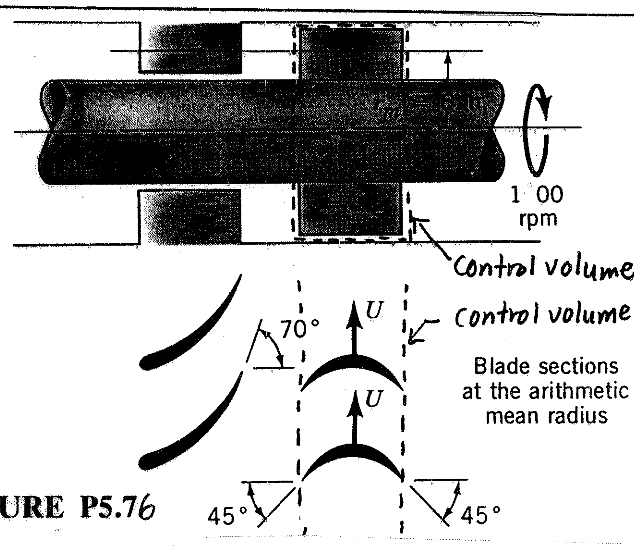
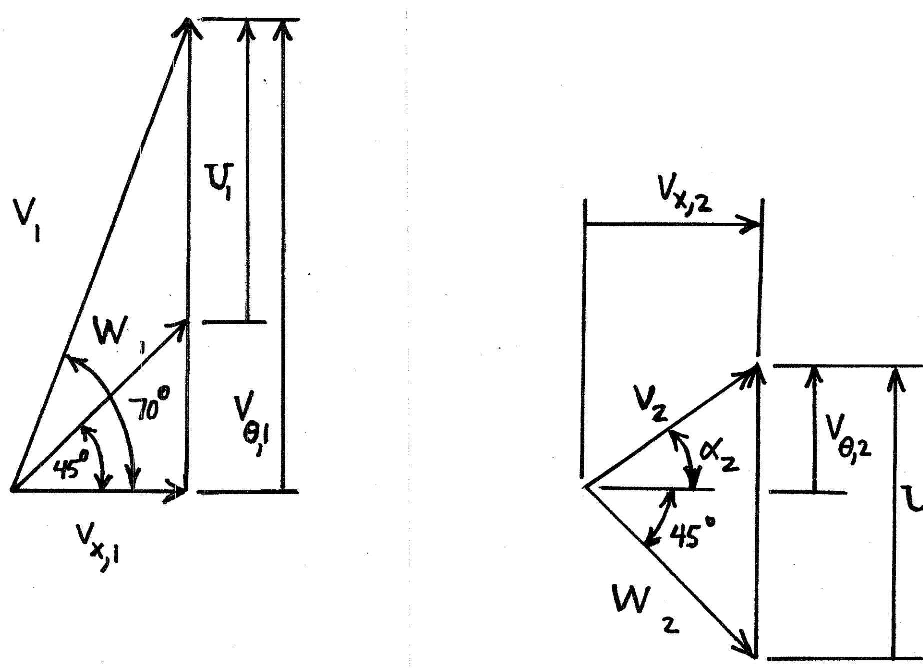



**FIGURE P5.76**

The velocity triangles for the flow entering and the flow leaving the rotor row at the arithmetic mean radius are sketched below.




At the arithmetic mean radius, the blade velocity, $U$, is

$$U_1 = U_2 = r_m \omega = \frac{(6 \text{ in.})\left(1000 \frac{\text{rev}}{\text{min}}\right)\left(2\pi \frac{\text{rad}}{\text{rev}}\right)}{\left(12 \frac{\text{in.}}{\text{ft}}\right)\left(60 \frac{\text{s}}{\text{min}}\right)} = 52.3 \frac{\text{ft}}{\text{s}}$$

With the velocity triangle for the flow entering the rotor we conclude that

$$V_1 \sin 70^\circ = V_{\theta,1} \quad (1)$$

$$V_1 \cos 70^\circ = V_{x,1} \quad (2)$$

$$W_1 \sin 45^\circ = V_{\theta,1} - U \quad (3)$$

$$W_1 \cos 45^\circ = V_{x,1} \quad (4)$$

(con't)

5.76 (con't)

From the ratio of Eqs. 3 and 4 we obtain

$$\tan 45^\circ = \frac{V_{\theta,1} - U}{V_{x,1}}$$

which when combined with Eqs. 1 and 2 yields

$$\tan 45^\circ = \frac{V_1 \sin 70^\circ - U}{V_1 \cos 70^\circ}$$

or

$$V_1 = \frac{U}{[\sin 70^\circ - (\cos 70^\circ)(\tan 45^\circ)]} = \frac{52.3 \frac{ft}{s}}{[\sin 70^\circ - (\cos 70^\circ)(\tan 45^\circ)]}$$

$$V_1 = 87.6 \frac{ft}{s}$$

Then

$$V_{\theta 1} = V_1 \sin 70^\circ = \left(87.6 \frac{ft}{s}\right) \sin 70^\circ = 82.3 \frac{ft}{s}$$

$$V_{x,1} = V_1 \cos 70^\circ = \left(87.6 \frac{ft}{s}\right) \cos 70^\circ = 29.9 \frac{ft}{s}$$

and

$$W_1 = \frac{V_{x,1}}{\cos 45^\circ} = \frac{\left(29.9 \frac{ft}{s}\right)}{\cos 45^\circ} = 42.4 \frac{ft}{s}$$

With the velocity triangle for the flow leaving the rotor we conclude that

$$W_2 \cos 45^\circ = V_{x,2} \qquad (5)$$

$$V_{\theta,2} = U_2 - W_2 \sin 45^\circ \qquad (6)$$

$$V_2 \sin \alpha_2 = V_{\theta,2} \qquad (7)$$

$$V_2 \cos \alpha_2 = V_{x,2} \qquad (8)$$

From the conservation of mass equation

$$V_{x,1} = V_{x,2} = 29.9 \frac{ft}{s}$$

(con't)

5.76 (con't)

Thus from Eq. 5

$$W_2 = \frac{V_{x,2}}{\cos 45°} = \frac{(29.9\ \frac{ft}{s})}{\cos 45°} = 42.4\ \frac{ft}{s}$$

and from Eq. 6

$$V_{\theta,2} = U_2 - W_2 \sin 45° = 52.3\ \frac{ft}{s} - (42.4\ \frac{ft}{s})\sin 45° = 22.4\ \frac{ft}{s}$$

The ratio of Eqs. 7 and 8 yields

$$\alpha_2 = \tan^{-1}\left(\frac{V_{\theta,2}}{V_{x,2}}\right) = \tan^{-1}\left[\frac{(22.4\ \frac{ft}{s})}{(29.9\ \frac{ft}{s})}\right] = 37°$$

and from Eq. 7

$$V_2 = \frac{V_{\theta,2}}{\sin \alpha_2} = \frac{(22.4\ \frac{ft}{s})}{\sin(37°)} = 37.2\ \frac{ft}{s}$$

We can use Eq. 5.54 to calculate the work per unit mass delivered at the shaft. Thus

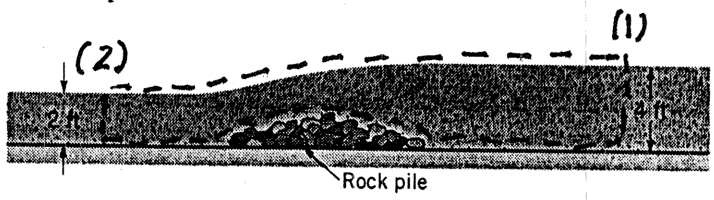
$$w_{shaft} = -U_1 V_{\theta,1} + U_2 V_{\theta,2}$$

$$w_{shaft} = \left[-(52.3\ \frac{ft}{s})(82.3\ \frac{ft}{s}) + (52.3\ \frac{ft}{s})(22.4\ \frac{ft}{s})\right]\left(\frac{1\ lb}{slug \cdot \frac{ft}{s^2}}\right)$$

$$w_{shaft} = -\underline{\underline{3130}}\ \frac{ft \cdot lb}{slug}$$

5.87

5.87 A 100-ft-wide river with a flowrate of 2400 $ft^3/s$ flows over a rock pile as shown in Fig. P5.87. Determine the direction of flow and the head loss associated with the flow across the rock pile.




■ FIGURE P5.87

To determine the direction of flow we will assume a direction, use the energy equation (Eq. 5.84) and calculate the head loss. If the head loss is positive, our assumed direction of flow is correct. If the head loss is negative which is not physically possible, our assumed direction of flow is wrong.

So, assuming the flow is from right to left or from point (1) to point (2) in the sketch above, we get using Eq. 5.84

$$\underbrace{\cancel{\frac{p_2}{\rho}} + \frac{V_2^2}{2g} + z_2 = \cancel{\frac{p_1}{\rho}}}_{\text{same pressure}} + \frac{V_1^2}{2g} + z_1 + \cancel{h_s}^{\,0,\text{ no shaft work}} - h_L$$

Now
$$V_1 = \frac{Q}{A_1} = \frac{\left(2400\,\frac{ft^3}{s}\right)}{(4\,ft)(100\,ft)} = 6\,\frac{ft}{s}$$

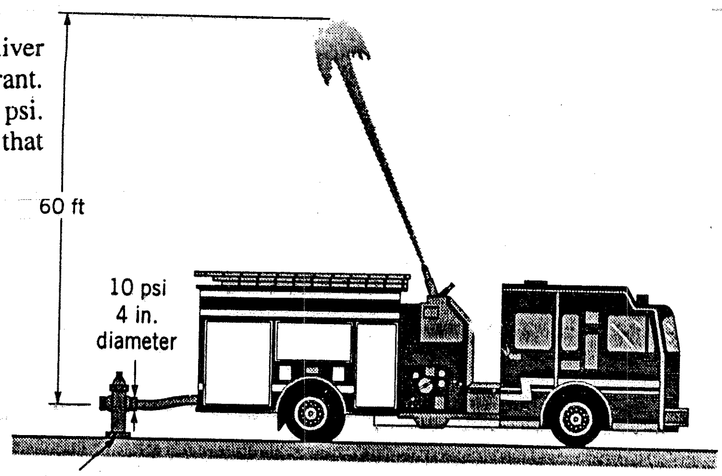
and
$$V_2 = \frac{Q}{A_2} = \frac{\left(2400\,\frac{ft^3}{s}\right)}{(2\,ft)(100\,ft)} = 12\,\frac{ft}{s}$$

So
$$h_L = \frac{V_1^2}{2g} - \frac{V_2^2}{2g} + z_1 - z_2 = \frac{\left(6\,\frac{ft}{s}\right)^2}{2\left(32.2\,\frac{ft}{s^2}\right)} - \frac{\left(12\,\frac{ft}{s}\right)^2}{2\left(32.2\,\frac{ft}{s^2}\right)} + 4\,ft - 2\,ft$$

$h_L = \underline{\underline{0.32\ ft}}$ and since $h_L$ is positive, our assumed <u>right to left</u> flow is correct

5.107

5.107 The pumper truck shown in Fig. P5.107 is to deliver 1.5 $ft^3/s$ to a maximum elevation of 60 ft above the hydrant. The pressure at the 4-in. diameter outlet of the hydrant is 10 psi. If head losses are negligibly small, determine the power that the pump must add to the water.




■ FIGURE P5.107

To solve this problem we first use the energy equation (Eq. 5.84) for flow from the hydrant exit (1) to the maximum desired elevation of 60 ft (2) to get $h_s$ or in this case, the pump head. With the pump head we can get the pump power from Eq. 5.85.

$$\cancelto{0}{\frac{p_2}{\rho}} + \cancelto{0}{\frac{V_2^2}{2g}} + z_2 = \frac{p_1}{\rho} + \frac{V_1^2}{2g} + z_1 + h_s - \cancelto{0}{h_L}$$

$$h_s = z_2 - z_1 - \frac{p_1}{\rho} - \frac{V_1^2}{2g}$$

$$V_1 = \frac{Q}{A_1} = \frac{Q}{\left(\frac{\pi d_1^2}{4}\right)} = \frac{\left(1.5\ \frac{ft^3}{s}\right)(4)}{\pi\left(\frac{4\ in.}{12\ \frac{in.}{ft}}\right)^2} = 17.2\ \frac{ft}{s}$$

$$h_s = 60\ ft - \frac{\left(10\ \frac{lb}{in.^2}\right)\left(144\ \frac{in.^2}{ft^2}\right)}{\left(62.4\ \frac{lb}{ft^3}\right)} - \frac{\left(17.2\ \frac{ft}{s}\right)^2}{2\left(32.2\ \frac{ft}{s^2}\right)}$$

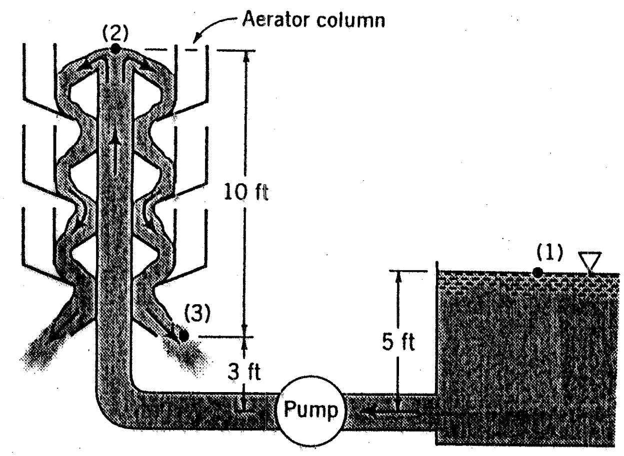
$$h_s = 32.3\ ft$$

$$\dot{W}_{\substack{shaft\\net\ in}} = \gamma Q h_s = \left(62.4\ \frac{lb}{ft^3}\right)\left(1.5\ \frac{ft^3}{s}\right)\frac{(32.2\ ft)}{\left(550\ \frac{ft.lb}{s.hp}\right)}$$

$$\dot{W}_{\substack{shaft\\net\ in}} = \underline{\underline{5.48\ hp}}$$

5.118

**5.118** Water is pumped from a tank, point (1), to the top of a water plant aerator, point (2), as shown in **Video V5.8** and **Fig.** P5.118 at a rate of 3.0 $ft^3/s$. **(a)** Determine the power that the pump adds to the water if the head loss from (1) to (2) where $V_2 = 0$ is 4 ft. **(b)** Determine the head loss from (2) to the bottom of the aerator column, point (3), if the average velocity at (3) is $V_3 = 2$ ft/s.




■ FIGURE P5.118

(a) The energy equation from (1) to (2)

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p - h_L = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

with

$p_1 = p_2 = V_1 = V_2 = 0$ gives

$$h_p = h_L + z_2 - z_1 = 4\,ft + (10+3)\,ft - 5\,ft = 12\,ft$$

Thus, the pump power is

$$\dot{W}_s = \gamma Q h_s = 62.4\,\frac{lb}{ft^3}\left(3\,\frac{ft^3}{s}\right)(12\,ft) = 2246\,\frac{ft \cdot lb}{s}\left(\frac{1\,hp}{550\,\frac{ft\cdot lb}{s}}\right)$$

$$= \underline{\underline{4.08\,hp}}$$

(b) The energy equation from (2) to (3)

$$\frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_p - h_L = \frac{p_3}{\gamma} + \frac{V_3^2}{2g} + z_3$$

with

$p_2 = p_3 = V_2 = h_p = 0$ gives

$$h_L = z_2 - z_3 - \frac{V_3^2}{2g} = 13\,ft - 3\,ft - \frac{\left(2\,\frac{ft}{s}\right)^2}{2\left(32.2\,\frac{ft}{s^2}\right)} = 10\,ft - 0.062\,ft$$

or

$$h_L = \underline{\underline{9.94\,ft}}$$