PI/PID Controller Design Based on Direct Synthesis and Disturbance Rejection

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A design method for PID controllers based on the direct synthesis approach and specification of the desired closed-loop transfer function for disturbances is proposed. Analytical expressions for PID controllers are derived for several common types of process models, including first-order and second-order plus time delay models and an integrator plus time delay model. Although the controllers are designed for disturbance rejection, the set-point responses are usually satisfactory and can be tuned independently via a set-point weighting factor. Nine simulation examples demonstrate that the proposed design method results in very good control for a wide variety of processes including those with integrating and/or nonminimum phase characteristics. The simulations show that the proposed design method provides better disturbance rejection than the standard direct synthesis and internal model control methods when the controllers are tuned to have the same degree of robustness.

1. Introduction

The ubiquitous PID controller has continued to be the most widely used process control technique for many decades. Although advanced control techniques such as model predictive control can provide significant improvements, a PID controller that is properly designed and tuned has proved to be satisfactory for the vast majority of industrial control loops.[1,2] The enormous literature on PID controllers includes a wide variety of design and tuning methods based on different performance criteria.[3−6] Two early and well-known design methods were reported by Ziegler and Nichols (ZN)[7] and Cohen and Coon.[8] Both methods were developed to provide a closed-loop response with a quarter decay ratio. Other well-known formulas for PI controller design include design relations based on integral error criteria[9−11] and gain and phase margin formulas.[12]

The design methods for PID controllers are typically based on a time-domain or frequency-domain performance criterion. However, the relationships between the dynamic behavior of the closed-loop system and these performance indices are not straightforward. In the direct synthesis (DS) approach,[13−15] however, the controller design is based on a desired closed-loop transfer function. Then, the controller is calculated analytically so that the closed-loop set-point response matches the desired response. The obvious advantage of the direct synthesis approach is that performance requirements are incorporated directly through specification of the closed-loop transfer function. One way to specify the closed-loop transfer function is to choose the closed-loop poles. This pole placement method[4,13] can be interpreted as a special type of direct synthesis.

In general, controllers designed using the DS method do not necessarily have a PID control structure. However, a PI or PID controller can be derived for simple process models such as first- or second-order plus time delay models by choosing appropriate closed-loop transfer functions.[15,16] For example, the λ-tuning method was originally proposed by Dahlin[17] and is widely used in the process industries. It is based on a first-order plus time delay model that has a relatively large time delay. The resulting controller is a PI controller with time-delay compensation.[4] Also, the well-known internal model control (IMC) design method[1,18−20] is closely related to the DS method and produces identical PID controllers for a wide range of problems. For higher-order systems, a model reduction technique and IMC can be used to synthesize PID controllers.[21] Alternatively, a high-order controller can be designed and then reduced to PID form by a series expansion.[22]

DS design methods are usually based on specification of the desired closed-loop transfer function for set-point changes. Consequently, the resulting DS controllers tend to perform well for set-point changes, but the disturbance response might not be satisfactory. For example, the IMC−PID controller provides good set-point tracking but very sluggish disturbance responses for processes with a small time-delay/time-constant ratio.[1] However, for many process control applications, disturbance rejection is much more important than set-point tracking. Therefore, controller design that emphasizes disturbance rejection, rather than set-point tracking, is an important design problem that has received renewed interest recently.

Middleton and Graebe[23] have investigated the relationship between input disturbance responses and robustness. They concluded that the decision to cancel, rather than shift, slow stable open-loop poles involves a design tradeoff between input disturbance rejection and robustness. Lee et al.[22] extended the IMC design approach for two degree of freedom controllers to improve disturbance performance. Their controller is a combination of two controllers, a standard IMC controller for set-point changes and a second IMC type of controller designed to shape the disturbance response. Their control system also includes a set-point filter that is specified as the inverse of the IMC controller for disturbances. This design provides a set-point response that is identical to that for the standard IMC controller. This novel control scheme can provide improved dy-

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10.1021/ie010756m CCC: $22.00
Published on Web 08/16/2002

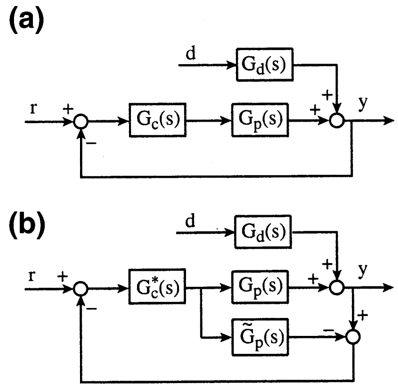

Figure 1. Feedback control strategies. (a) Classical feedback control. (b) Internal model control.

namic performance over standard IMC controllers, but the design procedure is more complicated and might result in unstable controllers.

It is somewhat surprising that the development of direct synthesis design methods for disturbance rejection has received relatively little attention. Early design methods for sampled-data systems were based on specifying the z transform of the desired closed-loop response to a particular disturbance.[14] However, this approach is sensitive to the assumed disturbance and does not necessarily produce a PID controller. A more promising approach was proposed recently by Szita and Sanathanan.[24−26] They specify the desired disturbance rejection characteristics in terms of a closed-loop transfer function for disturbances. The resulting controller usually is not a PI or PID controller and might be of high order. The authors propose approximating the high-order controller by a low-order controller using error minimization in the frequency domain.

In this paper, analytical expressions for PI and PID controllers are derived for common process models through the direct synthesis method and disturbance rejection. The proposed design method has a single design parameter, the desired closed-loop time constant, τ_c. The performance−robustness tradeoff involved in specifying τ_c is analyzed. A simple set-point weighting factor is used to improve controller performance for set-point changes without affecting the response to disturbances. Simulation results for nine examples demonstrate that the proposed design method provides robust PID controllers that perform well for both disturbance and set-point changes.

2. Direct Synthesis Method Based on Set-Point Responses

In the direct synthesis approach, an analytical expression for the feedback controller is derived from a process model and a desired closed-loop response. In most of the DS literature, the desired closed-loop response is expressed as a closed-loop transfer function for set-point changes. Consequently, this popular version of the direct synthesis method will be briefly introduced in the next section.

2.1. Direct Synthesis for Set-Point Tracking (DS). Consider a feedback control system with the standard block diagram in Figure 1a. Assume that $G_p(s)$ is a model of the process, measuring element, transmitter, and control valve.

The closed-loop transfer function for set-point changes is derived as

$$\frac{y}{r} = \frac{G_p(s)\, G_c(s)}{1 + G_p(s)\, G_c(s)} \tag{1}$$

Rearranging gives an expression for the feedback controller

$$G_c(s) = \frac{\left(\frac{y}{r}\right)}{G_p(s)\left[1 - \left(\frac{y}{r}\right)\right]} \tag{2}$$

Let the desired closed-loop transfer function for set-point changes be specified as $(y/r)_d$, and assume that a process model $\tilde{G}_p(s)$ is available. Replacing the unknown (y/r) and $G_p(s)$ by $(y/r)_d$ and $\tilde{G}_p(s)$, respectively, gives a design equation for $G_c(s)$

$$G_c(s) = \frac{\left(\frac{y}{r}\right)_d}{\tilde{G}_p(s)\left[1 - \left(\frac{y}{r}\right)_d\right]} \tag{3}$$

Because the characteristics of $(y/r)_d$ have a direct impact on the resulting controller, $(y/r)_d$ should be chosen so that the closed-loop performance is satisfactory and the resulting controller is physically realizable.

The DS controller in eq 3 results in the following closed-loop transfer functions

$$\left(\frac{y}{r}\right)^{DS} = \frac{G_p\left(\frac{y}{r}\right)_d}{\tilde{G}_p + \left(\frac{y}{r}\right)_d (G_p - \tilde{G}_p)} \tag{4}$$

$$\left(\frac{y}{d}\right)^{DS} = \frac{\tilde{G}_p G_d\left[1 - \left(\frac{y}{r}\right)_d\right]}{\tilde{G}_p + \left(\frac{y}{r}\right)_d (G_p - \tilde{G}_p)} \tag{5}$$

For the ideal case where the process model is perfect (i.e., $G_p = \tilde{G}_p$), the closed-loop transfer functions become

$$\left(\frac{y}{r}\right)^{DS} = \left(\frac{y}{r}\right)_d \tag{6}$$

$$\left(\frac{y}{d}\right)^{DS} = G_d\left[1 - \left(\frac{y}{r}\right)_d\right] \tag{7}$$

respectively.

2.2. Comparison with Internal Model Control (IMC). A well-known control system design strategy, internal model control (IMC) was developed by Morari and co-workers[20] and is closely related to the direct synthesis approach. Like the DS method, the IMC method is based on an assumed process model and relates the controller settings to the model parameters in a straightforward manner. The IMC approach has the advantages that it makes the consideration of model uncertainty and the making of tradeoffs between control system performance and robustness easier.

The IMC approach has the simplified block diagram shown in Figure 1b, where $\tilde{G}_p(s)$ is the process model and $G_c^*(s)$ is the IMC controller. The IMC controller design involves two steps:

Step 1. The process model $\tilde{G}_p(s)$ is factored as

$$\tilde{G}_p(s) = \tilde{G}_{p+}(s)\, \tilde{G}_{p-}(s) \tag{8}$$

where $\tilde{G}_{p+}(s)$ contains any time delays and right-half-plane zeros. It is specified so that its steady-state gain is 1.

Step 2. The IMC controller is specified as

$$G_c^*(s) = \frac{1}{\tilde{G}_{p-}(s)} f \tag{9}$$

where f is a low-pass filter with a steady-state gain of 1. The IMC filter f typically has the form

$$f = \frac{1}{(\tau_c s + 1)^r} \tag{10}$$

where τ_c is the desired closed-loop time constant. Parameter r is a positive integer that is selected so that either G_c^* is a proper transfer function or the order of its numerator exceeds the order of the denominator by 1, if ideal derivative action is allowed.

The IMC structure, Figure 1b, can be converted into the conventional feedback control structure, Figure 1a.[27] Comparing the resulting controllers and the closed-loop responses of the IMC and direct synthesis (DS) approaches, it is obvious that these two approaches produce equivalent controllers and identical closed-loop performances in certain situations. For example, if the desired closed-loop response for set-point change is specified as $(y/r)_d = \tilde{G}_{p+} f$, then the DS controller is equivalent to the IMC controller, and identical closed-loop performance results, even when modeling errors are present.

2.3. Direct Synthesis for PI/PID Controllers. In general, both the direct synthesis and IMC methods do not necessarily result in PI/PID controllers. However, by choosing the appropriate desired closed-loop response and using either a Padé approximation or a power-series approximation for the time delay, PI/PID controllers can be derived for process models that are commonly used in industrial applications.

Choose the desired closed-loop transfer function as

$$\left(\frac{y}{r}\right)_d = \frac{e^{-\theta s}}{\tau_c s + 1} \tag{11}$$

where θ is the time delay of the system and τ_c is the design parameter. Then, the DS design eq 3 and a truncated power-series expansion for the time delay term in the denominator, $e^{-\theta s} \approx 1 - \theta s$, gives

$$G_c = \frac{1}{\tilde{G}_p} \frac{e^{-\theta s}}{(\tau_c + \theta)s} \tag{12}$$

For systems that can be described by first-order and second-order plus time delay models, a PI or PID controller can be obtained from eq 12. For a first-order plus time delay model

$$\tilde{G}_p(s) = \frac{K e^{-\theta s}}{\tau s + 1} \tag{13}$$

eq 12 reduces to

$$G_c = \frac{\tau s + 1}{K(\tau_c + \theta)s} \tag{14}$$

Equation 14 can be expressed as an ideal PI controller

$$G_{PI}(s) = K_c\left(1 + \frac{1}{\tau_I s}\right) \tag{15}$$

with the following controller settings

$$K_c = \frac{1}{K} \frac{\tau}{\tau_c + \theta} \tag{16}$$

$$\tau_I = \tau \tag{17}$$

For a second-order plus time delay model

$$\tilde{G}_p(s) = \frac{K e^{-\theta s}}{(\tau_1 s + 1)(\tau_2 s + 1)} \tag{18}$$

substituting into eq 12 gives an ideal PID controller

$$G_{PID}(s) = K_c\left(1 + \frac{1}{\tau_I s} + \tau_D s\right) \tag{19}$$

with the following settings

$$K_c = \frac{1}{K} \frac{\tau_1 + \tau_2}{\theta + \tau_c} \tag{20}$$

$$\tau_I = \tau_1 + \tau_2 \tag{21}$$

$$\tau_D = \frac{\tau_1 \tau_2}{\tau_1 + \tau_2} \tag{22}$$

Identical PI/PID settings have been obtained using the IMC approach.[1,18]

3. Direct Synthesis Design for Disturbance Rejection

The PI/PID settings obtained from the DS and IMC approaches are based on specifying the closed-loop transfer function for set-point changes. For processes with small time-delay/time-constant ratios, these PI/PID controllers provide very sluggish disturbance responses.[1] Therefore, it is worthwhile to develop a modified direct synthesis approach based on disturbance rejection. The new design method will be denoted by “DS-d”.

Consider a control system with the standard block diagram shown in Figure 1a. The closed-loop transfer function for disturbances is given by

$$\frac{y}{d} = \frac{G_d(s)}{1 + G_p(s)\, G_c(s)} \tag{23}$$

Rearranging gives an expression for the feedback controller

$$G_c(s) = \frac{G_d(s)}{\left(\frac{y}{d}\right) G_p(s)} - \frac{1}{G_p(s)} \tag{24}$$

Let the desired closed-loop transfer function for disturbances be specified as $(y/d)_d$, and assume that a process model $\tilde{G}_p(s)$ and a disturbance model $\tilde{G}_d(s)$ are available. Replacing the unknown (y/d), $G_p(s)$, and $G_d(s)$ by $(y/d)_d$, $\tilde{G}_p(s)$, and $\tilde{G}_d(s)$, respectively, gives a design equation for $G_c(s)$

$$G_c(s) = \frac{\tilde{G}_d(s)}{\left(\frac{y}{d}\right)_d \tilde{G}_p(s)} - \frac{1}{\tilde{G}_p(s)} \quad (25)$$

For the DS-d controller in eq 25, the closed-loop transfer functions are

$$\left(\frac{y}{r}\right)^{DS-d} = \frac{G_p\left[\tilde{G}_d - \left(\frac{y}{d}\right)_d\right]}{G_p\tilde{G}_d + \left(\frac{y}{d}\right)_d(\tilde{G}_p - G_p)} \quad (26)$$

$$\left(\frac{y}{d}\right)^{DS-d} = \frac{\tilde{G}_p G_d\left(\frac{y}{d}\right)_d}{G_p\tilde{G}_d + \left(\frac{y}{d}\right)_d(\tilde{G}_p - G_p)} \quad (27)$$

For the ideal case where the model is perfect (i.e., $\tilde{G}_p = G_p$ and $\tilde{G}_d = G_d$), the closed-loop transfer functions become

$$\left(\frac{y}{r}\right)^{DS-d} = 1 - \frac{\left(\frac{y}{d}\right)_d}{\tilde{G}_d(s)} \quad (28)$$

$$\left(\frac{y}{d}\right)^{DS-d} = \left(\frac{y}{d}\right)_d \quad (29)$$

respectively.

The DS-d design method does not necessarily produce a PI or PID controller. The structure and order of the controller depend on the specification of the desired closed-loop response and the process model. In this section, the new DS-d design method is used to derive PI/PID controllers for simple process models that are widely used.

3.1. DS-d PI Settings. The DS-d methods will now be used to design PI controllers for a first-order plus time delay model and then an integrator plus time delay model.

3.1.1. First-Order Plus Time Delay Model. Assume that the process is described by a first-order plus time-delay model

$$G_p(s) = \frac{Ke^{-\theta s}}{\tau s + 1} \quad (30)$$

and that $G_d(s) = G_p(s)$. (This latter assumption will be removed in section 3.3). Thus, if the PI controller in eq 15 is used, the closed-loop transfer function in eq 23 can be expressed as

$$\frac{y}{d} = \frac{\frac{Ke^{-\theta s}}{\tau s + 1}}{1 + \frac{Ke^{-\theta s}}{\tau s + 1}K_c\left(1 + \frac{1}{\tau_I s}\right)} \quad (31)$$

Approximating the time delay term in the denominator by a first-order power-series expansion, $e^{-\theta s} \approx 1 - \theta s$, and rearranging gives

$$\left(\frac{y}{d}\right) \approx \frac{\frac{\tau_I}{K_c}se^{-\theta s}}{\left(\frac{\tau}{K}\frac{\tau_I}{K_c} - \tau_I\theta\right)s^2 + \left(\frac{\tau_I}{KK_c} + \tau_I - \theta\right)s + 1} \quad (32)$$

Therefore, for PI controller design, it is reasonable to specify the desired closed-loop transfer function as

$$\left(\frac{y}{d}\right)_d = \frac{K_d se^{-\theta s}}{(\tau_c s + 1)^2} \quad (33)$$

with

$$K_d = \frac{\tau_I}{K_c} \quad (34)$$

The DS-d design equation, eq 25, produces a standard PI controller if the time delay in the denominator is approximated by, $e^{-\theta s} \approx 1 - \theta s$. The resulting PI controller parameters are

$$K_c = \frac{1}{K}\frac{\tau\theta + 2\tau\tau_c - \tau_c^2}{(\tau_c + \theta)^2} \quad (35)$$

$$\tau_I = \frac{\tau\theta + 2\tau\tau_c - \tau_c^2}{\tau + \theta} \quad (36)$$

and K_d is given by

$$K_d = K\frac{(\tau_c + \theta)^2}{\tau+\theta} \quad (37)$$

From eqs 35 and 36, it is apparent that, for large values of τ_c, anomalous results can occur because K_c and K can have opposite signs and τ_I can become negative. Both of these undesirable situations can be avoided, however, if the design parameter τ_c satisfies

$$0 < \tau_c < \tau + \sqrt{\tau^2 + \tau\theta} \quad (38)$$

This constraint on τ_c is not restrictive at all because direct synthesis controllers are typically designed so that $\tau_c < 2\tau$.

For this PI controller and $\tilde{G}_p = G_p$, the closed-loop transfer function for set-point changes is

$$\left(\frac{y}{r}\right)^{DS-d} \approx \frac{\tau_I s + 1}{(\tau_c s + 1)^2}e^{-\theta s} \quad (39)$$

3.1.2. Integrator Plus Time Delay Model. Processes with integrating characteristics are quite common in the process industries. Assume that the process is described by

$$G_p(s) = \frac{Ke^{-\theta s}}{s} \quad (40)$$

and that $G_d(s) = G_p(s)$. If a PI controller is used, the closed-loop transfer function for disturbances in eq 23 becomes

$$\frac{y}{d} = \frac{\frac{Ke^{-\theta s}}{s}}{1 + \frac{Ke^{-\theta s}}{s}K_c\left(1 + \frac{1}{\tau_I s}\right)} \quad (41)$$

Approximating the time delay term in the denominator by $e^{-\theta s} \approx 1 - \theta s$ gives

$$\left(\frac{y}{d}\right) \approx \frac{\dfrac{\tau_I}{K_c} s e^{-\theta s}}{\left(\dfrac{\tau_I}{KK_c} - \theta\tau_I\right)s^2 + (\tau_I - \theta)s + 1} \tag{42}$$

Therefore, if the desired closed-loop transfer function for disturbances is specified as

$$\left(\frac{y}{d}\right)_d = \frac{K_d s e^{-\theta s}}{(\tau_c s + 1)^2} \tag{43}$$

with $K_d = \tau_I/K_c$, then the controller obtained using the DS-d method can be rearranged to give a standard PI controller with

$$K_c = \frac{1}{K}\frac{2\tau_c + \theta}{(\tau_c + \theta)^2} \tag{44}$$

$$\tau_I = 2\tau_c + \theta \tag{45}$$

and K_d is given by

$$K_d = K(\tau_c + \theta)^2 \tag{46}$$

For this controller and $\tilde{G}_p = G_p$, the closed-loop response for set-point changes is

$$\left(\frac{y}{r}\right)^{DS-d} \approx \frac{\tau_I s + 1}{(\tau_c s + 1)^2} e^{-\theta s} \tag{47}$$

3.2. DS-d PID Settings. In this section, the proposed DS-d method is used to design PID controllers for some commonly used process models, including first-order and second-order plus time delay models and an integrator plus time delay model.

3.2.1. First-Order Plus Time Delay Model. Assume that the process is described by a first-order plus time delay model

$$G_p(s) = \frac{K e^{-\theta s}}{\tau s + 1} \tag{48}$$

and that $G_d(s) = G_p(s)$. Thus, if the PID controller in eq 19 is used, the closed-loop transfer function in eq 23 can be expressed as

$$\frac{y}{d} = \frac{\dfrac{K e^{-\theta s}}{\tau s + 1}}{1 + \dfrac{K e^{-\theta s}}{\tau s + 1} K_c\left(1 + \dfrac{1}{\tau_I s} + \tau_D s\right)} \tag{49}$$

Approximating the time delay term in the denominator by a first-order Padé approximation

$$e^{-\theta s} \approx \frac{1 - \dfrac{\theta}{2}s}{1 + \dfrac{\theta}{2}s} \tag{50}$$

and rearranging gives

$$\left(\frac{y}{d}\right) \approx \left[\frac{\tau_I}{K_c}s\left(1 + \frac{\theta}{2}s\right) e^{-\theta s}\right] \Big/ \left[\left(\frac{\theta\tau\tau_I}{2KK_c} - \frac{\theta}{2}\tau_I\tau_D\right)s^3 + \left(\frac{(\tau + \theta/2)\tau_I}{KK_c} + \tau_I\tau_D - \frac{\theta}{2}\tau_I\right)s^2 + \left(\frac{\tau_I}{KK_c} + \tau_I - \frac{\theta}{2}\right)s + 1\right] \tag{51}$$

Thus, for PID controller design, it is reasonable to specify the desired closed-loop transfer function as

$$\left(\frac{y}{d}\right)_d = \frac{K_d s\left(1 + \dfrac{\theta}{2}s\right)e^{-\theta s}}{(\tau_c s + 1)^3} \tag{52}$$

with $K_d = \tau_I/K_c$. Then, the controller obtained from the DS-d method, eq 25, can be rearranged to give a standard PID controller. The resulting PID controller parameters are

$$K_c = \frac{1}{K}\frac{\left(2\tau\theta + \dfrac{\theta^2}{2}\right)\left(3\tau_c + \dfrac{\theta}{2}\right) - 2\tau_c^{\ 3} - 3\tau_c^{\ 2}\theta}{2\tau_c^{\ 3} + 3\tau_c^{\ 2}\theta + \dfrac{\theta^2}{2}\left(3\tau_c + \dfrac{\theta}{2}\right)} \tag{53}$$

$$\tau_I = \frac{\left(2\tau\theta + \dfrac{\theta^2}{2}\right)\left(3\tau_c + \dfrac{\theta}{2}\right) - 2\tau_c^{\ 3} - 3\tau_c^{\ 2}\theta}{(2\tau + \theta)\theta} \tag{54}$$

$$\tau_D = \frac{3\tau_c^{\ 2}\tau\theta + \dfrac{\tau\theta^2}{2}\left(3\tau_c + \dfrac{\theta}{2}\right) - 2(\tau + \theta)\tau_c^{\ 3}}{\left(2\tau\theta + \dfrac{\theta^2}{2}\right)\left(3\tau_c + \dfrac{\theta}{2}\right) - 2\tau_c^{\ 3} - 3\tau_c^{\ 2}\theta} \tag{55}$$

and K_d is given by

$$K_d = K\frac{2\tau_c^{\ 3} + 3\tau_c^{\ 2}\theta + \dfrac{\theta^2}{2}\left(3\tau_c + \dfrac{\theta}{2}\right)}{(2\tau + \theta)\theta} \tag{56}$$

It is apparent from the above equations that KK_c, τ_I, and τ_D can be negative for large values of τ_c. However, simulation experience has demonstrated that this potential problem does not occur if the closed-loop time constant τ_c is chosen in a reasonable manner.

For this PID controller and $\tilde{G}_p = G_p$, the closed-loop transfer function for set-point changes is obtained as

$$\left(\frac{y}{r}\right)^{DS-d} \approx \frac{(\tau_I\tau_D s^2 + \tau_I s + 1)\left(1 + \dfrac{\theta}{2}s\right)}{(\tau_c s + 1)^3} e^{-\theta s} \tag{57}$$

3.2.2. Integrator Plus Time Delay Model. Now assume that the process is described by

$$G_p(s) = \frac{K e^{-\theta s}}{s} \tag{58}$$

and that $G_d(s) = G_p(s)$. If a PID controller is used, the closed-loop transfer function in eq 23 becomes

$$\frac{y}{d} = \frac{\dfrac{K e^{-\theta s}}{s}}{1 + \dfrac{K e^{-\theta s}}{s} K_c\left(1 + \dfrac{1}{\tau_I s} + \tau_D s\right)} \tag{59}$$

Approximating the time delay term in the denominator by eq 50 gives

$$\left(\frac{y}{d}\right) \approx \left[\frac{\tau_I}{K_c}s\left(1+\frac{\theta}{2}s\right)e^{-\theta s}\right] \Big/ \left[\left(\frac{\theta\tau_I}{2KK_c}-\frac{\theta}{2}\tau_I\tau_D\right)s^3 + \left(\frac{\tau_I}{KK_c}+\tau_I\tau_D-\frac{\theta}{2}\tau_I\right)s^2+\left(\tau_I-\frac{\theta}{2}\right)s+1\right] \quad (60)$$

Therefore, if the desired closed-loop transfer function for disturbances is specified as

$$\left(\frac{y}{d}\right)_d = \frac{K_d s\left(1+\frac{\theta}{2}s\right)e^{-\theta s}}{(\tau_c s+1)^3} \quad (61)$$

with $K_d = \tau_I/K_c$, then the controller obtained using the DS-d method can be rearranged to give a standard PID controller. The resulting PID controller parameters are

$$K_c = \frac{1}{K}\frac{\theta\left(3\tau_c+\frac{\theta}{2}\right)}{\left(\tau_c+\frac{\theta}{2}\right)^3} \quad (62)$$

$$\tau_I = 3\tau_c + \frac{\theta}{2} \quad (63)$$

$$\tau_D = \frac{\frac{3}{2}\tau_c^{\,2}\theta+\frac{3}{4}\tau_c\theta^2+\frac{\theta^3}{8}-\tau_c^{\,3}}{\theta\left(3\tau_c+\frac{\theta}{2}\right)} \quad (64)$$

and K_d is given by

$$K_d = K\frac{\left(\tau_c+\frac{\theta}{2}\right)^3}{\theta} \quad (65)$$

For this controller and $\tilde{G}_p = G_p$, the closed-loop response for set-point change is obtained as

$$\left(\frac{y}{r}\right)^{DS-d} \approx \frac{(\tau_I\tau_D s^2+\tau_I s+1)\left(1+\frac{\theta}{2}s\right)}{(\tau_c s+1)^3}e^{-\theta s} \quad (66)$$

3.2.3. Second-Order Plus Time Delay Model. Assume that the process is described by a second-order plus time delay model

$$G_p(s) = \frac{Ke^{-\theta s}}{(\tau_1 s+1)(\tau_2 s+1)} \quad (67)$$

and that $G_d(s) = G_p(s)$. Thus, if a PID controller is used, the closed-loop transfer function in eq 23 can be expressed as

$$\frac{y}{d} = \frac{\frac{Ke^{-\theta s}}{(\tau_1 s+1)(\tau_2 s+1)}}{1+\frac{Ke^{-\theta s}}{(\tau_1 s+1)(\tau_2 s+1)}K_c\left(1+\frac{1}{\tau_I s}+\tau_D s\right)} \quad (68)$$

Approximating the time delay term in the denominator by a first-order power-series expansion, $e^{-\theta s} \approx 1 - \theta s$, and rearranging gives

$$\left(\frac{y}{d}\right) \approx \left[\frac{\tau_I}{K_c}se^{-\theta s}\right] \Big/ \left[\left(\frac{\tau_1\tau_2\tau_I}{KK_c}-\theta\tau_I\tau_D\right)s^3 + \left(\frac{(\tau_1+\tau_2)\tau_I}{KK_c}+\tau_I\tau_D-\theta\tau_I\right)s^2+\left(\frac{\tau_I}{KK_c}+\tau_I-\theta\right)s+1\right] \quad (69)$$

Therefore, if the desired closed-loop transfer function for disturbance is specified as

$$\left(\frac{y}{d}\right)_d = \frac{K_d se^{-\theta s}}{(\tau_c s+1)^3} \quad (70)$$

with $K_d = \tau_I/K_c$, then the controller obtained from the DS-d method, eq 25, can be rearranged to give a standard PID controller. The resulting PID controller parameters are

$$K_c = \frac{1}{K}\frac{[(\tau_1+\tau_2)\theta+\tau_1\tau_2](3\tau_c+\theta)-\tau_c^{\,3}-3\tau_c^{\,2}\theta}{(\tau_c+\theta)^3} \quad (71)$$

$$\tau_I = \frac{[(\tau_1+\tau_2)\theta+\tau_1\tau_2](3\tau_c+\theta)-\tau_c^{\,3}-3\tau_c^{\,2}\theta}{\tau_1\tau_2+(\tau_1+\tau_2+\theta)\theta} \quad (72)$$

$$\tau_D = \frac{3\tau_c^{\,2}\tau_1\tau_2+\tau_1\tau_2\theta(3\tau_c+\theta)-(\tau_1+\tau_2+\theta)\tau_c^{\,3}}{[(\tau_1+\tau_2)\theta+\tau_1\tau_2](3\tau_c+\theta)-\tau_c^{\,3}-3\tau_c^{\,2}\theta} \quad (73)$$

and K_d is given by

$$K_d = K\frac{(\tau_c+\theta)^3}{\tau_1\tau_2+(\tau_1+\tau_2+\theta)\theta} \quad (74)$$

For this PID controller and $\tilde{G}_p = G_p$, the closed-loop transfer function for set-point changes is

$$\left(\frac{y}{r}\right)^{DS-d} \approx \frac{(\tau_I\tau_D s^2+\tau_I s+1)}{(\tau_c s+1)^3}e^{-\theta s} \quad (75)$$

3.2.4. First-Order with an Integrator Plus Time Delay Model. Assume that the process is described by

$$G_p(s) = \frac{Ke^{-\theta s}}{s(\tau s+1)} \quad (76)$$

and that $G_d(s) = G_p(s)$. If a PID controller is used, then the closed-loop transfer function in eq 23 becomes

$$\frac{y}{d} = \frac{\frac{Ke^{-\theta s}}{s(\tau s+1)}}{1+\frac{Ke^{-\theta s}}{s(\tau s+1)}K_c\left(1+\frac{1}{\tau_I s}+\tau_D s\right)} \quad (77)$$

Approximating the time delay term in the denominator by $e^{-\theta s} \approx 1 - \theta s$ gives

$$\left(\frac{y}{d}\right) \approx \left[\frac{\tau_I}{K_c}se^{-\theta s}\right] \Big/ \left[\left(\frac{\tau\tau_I}{KK_c}-\theta\tau_I\tau_D\right)s^3 + \left(\frac{\tau_I}{KK_c}+\tau_I\tau_D-\theta\tau_I\right)s^2+(\tau_I-\theta)s+1\right] \quad (78)$$

Therefore, if the desired closed-loop transfer function

Table 1. PI/PID Controller Settings for the DS-d Design Method

case[a,b]	model	KK_c	τ_I	τ_D
A	$\frac{Ke^{-\theta s}}{\tau s+1}$	$\frac{\tau^2+\tau\theta-(\tau_c-\tau)^2}{(\tau_c+\theta)^2}$	$\frac{\tau^2+\tau\theta-(\tau_c-\tau)^2}{\tau+\theta}$	–
B	$\frac{Ke^{-\theta s}}{\tau s+1}$	$\frac{\left(2\tau\theta+\frac{\theta^2}{2}\right)\left(3\tau_c+\frac{\theta}{2}\right)-2\tau_c^{\ 3}-3\tau_c^{\ 2}\theta}{2(\tau_c+\theta/2)^3}$	$\frac{\left(2\tau\theta+\frac{\theta^2}{2}\right)\left(3\tau_c+\frac{\theta}{2}\right)-2\tau_c^{\ 3}-3\tau_c^{\ 2}\theta}{(2\tau+\theta)\theta}$	$\frac{3\tau_c^{\ 2}\tau\theta+\frac{\tau\theta^2}{2}\left(3\tau_c+\frac{\theta}{2}\right)-2(\tau+\theta)\tau_c^{\ 3}}{\left(2\tau\theta+\frac{\theta^2}{2}\right)\left(3\tau_c+\frac{\theta}{2}\right)-2\tau_c^{\ 3}-3\tau_c^{\ 2}\theta}$
C	$\frac{Ke^{-\theta s}}{s}$	$\frac{2\tau_c+\theta}{(\tau_c+\theta)^2}$	$2\tau_c+\theta$	–
D	$\frac{Ke^{-\theta s}}{s}$	$\frac{\theta\left(3\tau_c+\frac{\theta}{2}\right)}{\left(\tau_c+\frac{\theta}{2}\right)^3}$	$3\tau_c+\frac{\theta}{2}$	$\frac{\left(\tau_c+\frac{\theta}{2}\right)^3-2\tau_c^{\ 3}}{\theta\left(3\tau_c+\frac{\theta}{2}\right)}$
E	$\frac{Ke^{-\theta s}}{s(\tau s+1)}$	$\frac{(3\tau_c+\theta)(\tau+\theta)}{(\tau_c+\theta)^3}$	$3\tau_c+\theta$	$\frac{3\tau_c^{\ 2}\tau+3\tau_c\tau\theta-\tau_c^{\ 3}+\tau\theta^2}{(3\tau_c+\theta)(\tau+\theta)}$
F	$\frac{K(\tau_a s+1)}{s(\tau s+1)}$	$\frac{(3\tau_c-\tau_a)(\tau-\tau_a)}{(\tau_c-\tau_a)^3}$	$3\tau_c-\tau_a$	$\frac{3\tau_c^{\ 2}\tau-3\tau_c\tau\tau_a-\tau_c^{\ 3}+\tau\tau_a^{\ 2}}{(3\tau_c-\tau_a)(\tau-\tau_a)}$
G	$\frac{Ke^{-\theta s}}{(\tau_1 s+1)(\tau_2 s+1)}$	$\frac{[(\tau_1+\tau_2)\theta+\tau_1\tau_2](3\tau_c+\theta)-\tau_c^{\ 3}-3\tau_c^{\ 2}\theta}{(\tau_c+\theta)^3}$	$\frac{[(\tau_1+\tau_2)\theta+\tau_1\tau_2](3\tau_c+\theta)-\tau_c^{\ 3}-3\tau_c^{\ 2}\theta}{\tau_1\tau_2+(\tau_1+\tau_2+\theta)\theta}$	$\frac{3\tau_c^{\ 2}\tau_1\tau_2+\tau_1\tau_2\theta(3\tau_c+\theta)-(\tau_1+\tau_2+\theta)\tau_c^{\ 3}}{[(\tau_1+\tau_2)\theta+\tau_1\tau_2](3\tau_c+\theta)-\tau_c^{\ 3}-3\tau_c^{\ 2}\theta}$
H	$\frac{Ke^{-\theta s}}{\tau^2 s+2\zeta\tau s+1}$	$\frac{(2\zeta\tau\theta+\tau^2)(3\tau_c+\theta)-\tau_c^{\ 3}-3\tau_c^{\ 2}\theta}{(\tau_c+\theta)^3}$	$\frac{(2\zeta\tau\theta+\tau^2)(3\tau_c+\theta)-\tau_c^{\ 3}-3\tau_c^{\ 2}\theta}{\tau^2+(2\zeta\tau+\theta)\theta}$	$\frac{3\tau_c^{\ 2}\tau^2+\tau^2\theta(3\tau_c+\theta)-(2\zeta\tau+\theta)\tau_c^{\ 3}}{(2\zeta\tau\theta+\tau^2)(3\tau_c+\theta)-\tau_c^{\ 3}-3\tau_c^{\ 2}\theta}$
I	$\frac{K(\tau_a s+1)}{(\tau_1 s+1)(\tau_2 s+1)}$	$\frac{3\tau_c^{\ 2}\tau_a+[\tau_1\tau_2-(\tau_1+\tau_2)\tau_a](3\tau_c-\tau_a)-\tau_c^{\ 3}}{(\tau_c-\tau_a)^3}$	$\frac{3\tau_c^{\ 2}\tau_a+[\tau_1\tau_2-(\tau_1+\tau_2)\tau_a](3\tau_c-\tau_a)-\tau_c^{\ 3}}{\tau_1\tau_2-(\tau_1+\tau_2-\tau_a)\tau_a}$	$\frac{(\tau_a-\tau_1-\tau_2)\tau_c^{\ 3}+3\tau_c^{\ 2}\tau_1\tau_2-\tau_1\tau_2\tau_a(3\tau_c-\tau_a)}{3\tau_c^{\ 2}\tau_a+[\tau_1\tau_2-(\tau_1+\tau_2)\tau_a](3\tau_c-\tau_a)-\tau_c^{\ 3}}$

[a] Cases A and C, $\left(\frac{y}{d}\right)_d = \frac{K_d s e^{-\theta s}}{(\tau_c s+1)^2}$; cases B and D, $\left(\frac{y}{d}\right)_d = \frac{K_d s\left(1+\frac{\theta}{2}s\right)e^{-\theta s}}{(\tau_c s+1)^3}$; cases E, G, and H, $\left(\frac{y}{d}\right)_d = \frac{K_d s e^{-\theta s}}{(\tau_c s+1)^3}$; cases F and I, $\left(\frac{y}{d}\right)_d = \frac{K_d s(\tau_a s+1)}{(\tau_c s+1)^3}$. [b] $K_d = \tau_I/K_c$.

for disturbance is specified as

$$\left(\frac{y}{d}\right)_d = \frac{K_d s e^{-\theta s}}{(\tau_c s+1)^3} \tag{79}$$

with $K_d = \tau_I/K_c$, then the controller obtained from the DS-d method can be rearranged to give a standard PID controller. The resulting PID controller parameters are

$$K_c = \frac{1}{K}\frac{(3\tau_c+\theta)(\tau+\theta)}{(\tau_c+\theta)^3} \tag{80}$$

$$\tau_I = 3\tau_c + \theta \tag{81}$$

$$\tau_D = \frac{3\tau_c^{\ 2}\tau + 3\tau_c\tau\theta + \tau\theta^2 - \tau_c^{\ 3}}{(3\tau_c+\theta)(\tau+\theta)} \tag{82}$$

and K_d is give by

$$K_d = K\frac{(\tau_c+\theta)^3}{\tau+\theta} \tag{83}$$

For this PID controller and $\tilde{G}_p = G_p$, the closed-loop transfer function for set-point changes is

$$\left(\frac{y}{r}\right)^{DS-d} \approx \frac{(\tau_I\tau_D s^2 + \tau_I s + 1)}{(\tau_c s+1)^3}e^{-\theta s} \tag{84}$$

3.3. Discussion. In the previous sections, the DS-d method has been used to design PI/PID controllers for widely used process models. The resulting PI/PID controller settings are shown in Table 1.

Remark 1. The only design parameter, τ_c, is directly related to the closed-loop time constant. As τ_c decreases, the closed-loop response becomes faster.

Remark 2. Larger values of τ_c give larger values of K_d because $K_d = \tau_I/K_c$.

For a unit step disturbance at the process input, Åström and Hägglund[4] derived the following relation for the integral error (IE) associated with PI/PID control

$$IE = \int_0^\infty [r(t) - y(t)]\, dt = \frac{\tau_I}{K_c} \tag{85}$$

Thus

$$IE = K_d \tag{86}$$

This expression means that smaller τ_c values provide smaller IE values for step disturbances.

Remark 3. Although smaller values of τ_c provide better performance for disturbance and set-point changes, the control system robustness is worse. Therefore, the system robustness should be considered when τ_c is being chosen.

Remark 4. The PI/PID tuning rules in Table 1 were derived based on the assumption that $G_d = G_p$. For the more general case where $G_d \neq G_p$, the desired closed-loop response for disturbances is specified as

$$\left(\frac{y}{d}\right)_{d}^{*} = \left(\frac{y}{d}\right)_{d} \frac{G_{d}}{G_{p}} \quad (87)$$

where $(y/d)_d$ is given in Table 1. Thus, the PI/PID settings and the closed-loop responses for set-point changes are the same as for the special case where $G_d = G_p$.

Remark 5. The proposed PI tuning rules for integrating processes are equivalent to the IMC PI settings of Chien and Fruehauf.[1]

3.4. Set-Point and Derivative Weighting. Equations 15 and 19 are conventional PI and PID controllers. A more flexible control structure that includes set-point weighting and derivative weighting is given by Åström and Hägglund[4]

$$u(t) = K_{c}\left([br(t) - y(t)] + \frac{1}{\tau_{I}}\int_{0}^{t}[r(\tau) - y(\tau)]\,d\tau + \tau_{D}\frac{d[cr(t) - y(t)]}{dt}\right) \quad (88)$$

where the set-point weighting coefficient b is bounded by $0 \le b \le 1$ and the derivative weighting coefficient c is also bounded by $0 \le c \le 1$. The overshoot for set-point changes decreases with increasing b.

The controllers obtained for different values of b and c respond to disturbances and measurement noise in the same way as conventional PI/PID controllers, i.e., different values of b and c do not change the closed-loop response for disturbances. Therefore, the same PI/PID tuning rules developed here using the DS-d method are also applicable for the modified PI/PID controller in eq 88. However, the set-point response does depend on the values of b and c. If set-point weighting and derivative weighting are used, the closed-loop transfer function for set-point changes is given by

$$\frac{y}{r} = \frac{c\tau_{I}\tau_{D}s^{2} + b\tau_{I}s + 1}{\tau_{I}\tau_{D}s^{2} + \tau_{I}s+1}\,\frac{G_{p}(s)\,G_{c}(s)}{1 + G_{p}(s)\,G_{c}(s)} \triangleq G_{sp}(s) \quad (89)$$

where $G_c(s)$ is the conventional PID controller given by eq 19.

4. Simulation Results

Several simulation examples are used to demonstrate the proposed PI/PID tuning rules for the DS-d method. In practice, the derivative weighting factor c is usually set to zero to avoid a large derivative kick. Thus, in this paper, c is chosen to be zero for all of the simulation examples. Furthermore, the PID controller is implemented in the widely used "parallel form"

$$G_{c}(s) = K_{c}\left(1 + \frac{1}{\tau_{I}s} + \frac{\tau_{D}s}{\alpha\tau_{D}s + 1}\right) \quad (90)$$

The derivative filter parameter α is specified as $\alpha = 0.1$. Other implementations of PID control, such as the series form, are also widely used. The controller settings for one form can easily be converted to other forms.[3]

For each example, the DS-d, DS, ZN, and/or some other methods were used to design PI/PID controllers. As mentioned in section 2.3, the DS and IMC methods can provide identical PI/PID settings if the same closed-loop transfer function is specified and the same approximation is used for time delay term. For some process models, however, the IMC tuning rules are very well-known. Thus, the IMC method was used instead of the DS method for a few examples.

Table 2. PID Controller Settings for Example 1 (θ/τ = 0.01)

tuning method	K_c	τ_I	τ_D	M_S	set point IAE	set point TV	disturbance IAE	disturbance TV
DS-d (τ_c = 1.2, b = 1)	0.829	4.05	0.354	1.94	3.06	1.46	4.89	1.89
DS-d (τ_c = 1.2, b = 0.5)	0.829	4.05	0.354	1.94	2.19	0.82	4.89	1.89
IMC (τ_c = 0.85)	0.744	100.5	0.498	1.94	1.88	1.10	84.4	1.59
ZN	0.948	1.99	0.498	2.30	3.52	2.82	3.22	3.06

The following robustness and performance metrics were used as evaluation criteria for the comparison of the PI/PID controllers:

Robustness Metric. The peak value of the sensitivity function, M_S,[28] has been widely used as a measure of system robustness. Recommended values of M_S are typically in the range of 1.2−2.0.[29]

To provide fair comparisons, the model-based controllers (DS-d, DS, and IMC) were tuned by adjusting τ_c so that the M_S values were very close. This tuning facilitated a comparison of controller performance for disturbance and set-point changes for controllers that had the same degree of robustness.

Performance Metrics. Two metrics were used to evaluate controller performance. The integrated absolute error (IAE) is defined as[15]

$$\text{IAE} \triangleq \int_{0}^{\infty}|\,r(t) - y(t)|\,dt \quad (91)$$

To evaluate the required control effort, the total variation (TV) of the manipulated input u was calculated

$$\text{TV} \triangleq \sum_{k=1}^{\infty}|u(k+1) - u(k)| \quad (92)$$

The total variation is a good measure of the "smoothness" of a signal and should be as small as possible.[30]

4.1. Example 1. Consider the following model with a step disturbance acting at the plant input[31]

$$G_{p}(s) = G_{d}(s) = \frac{100}{100s + 1}e^{-s} \quad (93)$$

The DS-d, IMC, and ZN methods were used to design the PID controllers shown in Table 2. For the DS-d method, a value of $\tau_c = 1.2$ was chosen so that $M_S = 1.94$. To obtain a fair comparison, $\tau_c = 0.85$ was selected for the IMC method so that $M_S = 1.94$.

The simulation results in Figure 2 and the IAE and TV values in Table 2 indicate that the disturbance response for the DS-d controller is much better and faster than the IMC response, whereas the movements of these two controller outputs are similar. The disturbance response of the ZN controller has a smaller peak value but is more oscillatory than the DS-d response. The set-point response of the DS-d controller is more sluggish and has a larger overshoot than the IMC controller. However, the overshoot for the DS-d controller can be eliminated, without affecting the disturbance response, by setting $b = 0.5$. By comparing the responses and the M_S, IAE, and TV values of all three controllers,

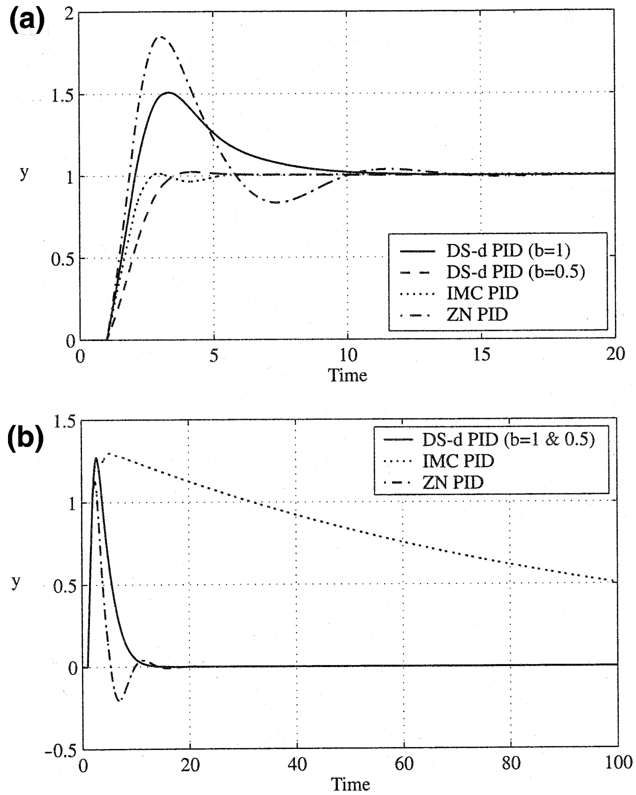

Figure 2. Simulation results of PID controllers for example 1 (θ/τ = 0.01). (a) Response to a unit step set-point change. (b) Response to a unit step disturbance.

Table 3. PI Controller Settings for Example 2 (θ/τ = 0.25)

				set point		disturbance	
tuning method	K_c	τ_I	M_S	IAE	TV	IAE	TV
DS-d (τ_c = 0.35, b = 1)	2.30	0.662	1.88	0.635	3.64	0.288	1.54
DS-d (τ_c = 0.35, b = 0.5)	2.30	0.662	1.88	0.630	2.10	0.288	1.54
DS (τ_c = 0.13)	2.63	1	1.90	0.532	3.80	0.37	1.40
ZN	3.12	0.763	2.37	0.632	6.63	0.244	2.02

it can be concluded that the DS-d controller provides the best performance without using excessive control effort.

4.2. Example 2. Consider a process is described by

$$G_p(s) = G_d(s) = \frac{e^{-0.25s}}{s+1} \tag{94}$$

The PI controller characteristics for the DS-d, DS, and ZN controllers are shown in Table 3.

Unit step changes were introduced in the set point (at t = 0) and in the disturbance (at t = 4 min). The simulation results in Figure 3 indicate that the disturbance response of the DS-d PI controller is faster than that of the DS PI controller and less oscillatory than that of the ZN PI controller. The set-point response of the DS-d PI controller exhibits overshoot, but it can be reduced by setting b = 0.5. These conclusions can be confirmed by the IAE and TV values in Table 3.

Figure 4 shows the simulation results for the practical situation where there are inequality constraints on the manipulated variable u: $-2 \le u \le 2$. A comparison of Figures 3 and 4 indicates that the initial set-point responses of the DS-d (b = 1), DS, and ZN PI controllers were slower because of controller saturation. Also, the oscillations for the DS and ZN PI controllers were damped. The disturbance responses were not affected because inequality constraints on u were not active.

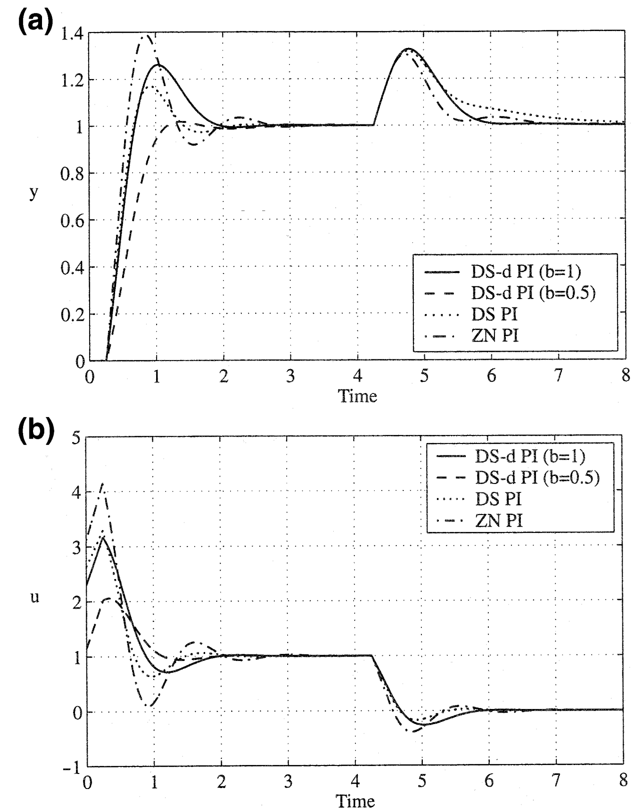

Figure 3. Simulation results without u constraints for example 2 (θ/τ = 0.25). (a) Controlled variable y. (b) Manipulated variable u.

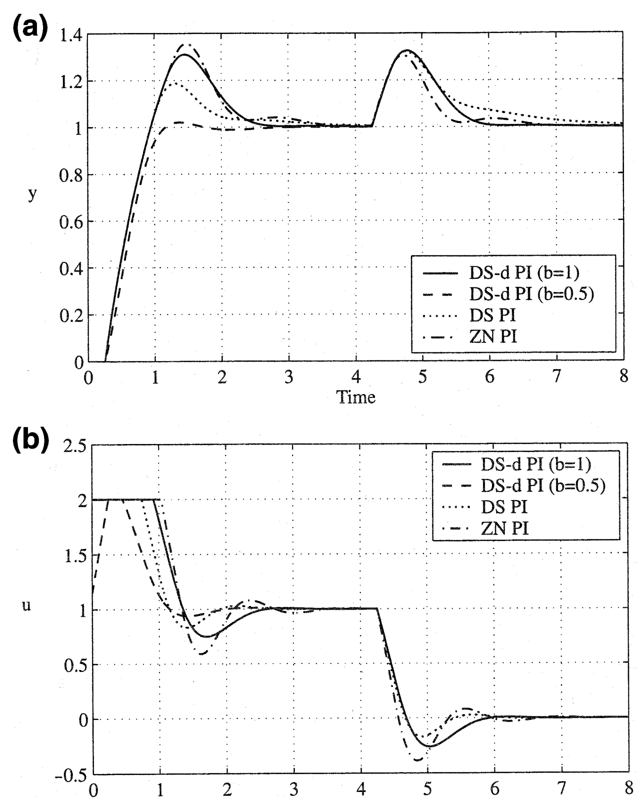

Figure 4. Simulation results with u constraints for example 2 (θ/τ = 0.25). (a) Controlled variable y. (b) Manipulated variable u.

Examples 1 and 2 demonstrate that the DS-d method provides better performance than the DS and IMC methods for processes with small θ/τ ratios. The follow-

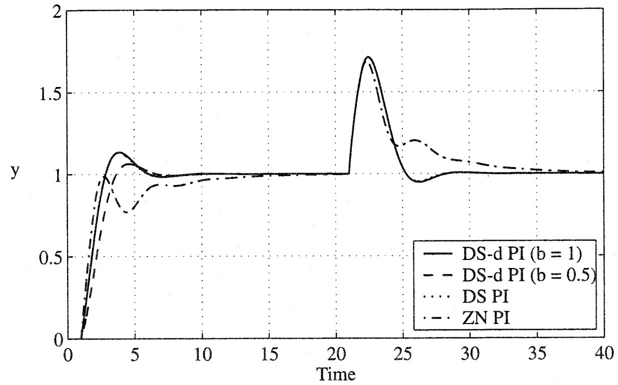

Figure 5. Simulation results of PI controllers for example 3 (θ/τ = 1).

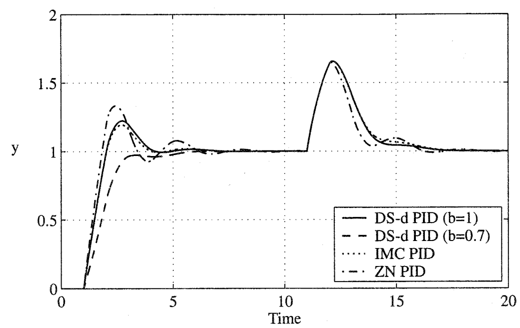

Figure 6. Simulation results of PID controllers for example 3 (θ/τ = 1).

Table 4. PI Controller Settings for Example 3 (θ/τ = 1)

				set point		disturbance	
tuning method	K_c	τ_I	M_S	IAE	TV	IAE	TV
DS-d (τ_c = 0.8, b = 1)	0.60	0.98	1.80	2.13	1.09	1.80	1.30
DS-d (τ_c = 0.8, b = 0.5)	0.60	0.98	1.80	2.34	1.09	1.80	1.30
DS (τ_c = 0.62)	0.62	1.00	1.81	2.11	1.09	1.80	1.30
ZN	1.02	2.58	2.05	2.52	1.62	2.50	1.45

Table 5. PID Controller Settings for Example 3 (θ/τ = 1)

					set point		disturbance	
tuning method	K_c	τ_I	τ_D	M_S	IAE	TV	IAE	TV
DS-d (τ_c = 0.75, b = 1)	1.11	1.45	0.317	1.92	1.68	2.18	1.30	1.46
DS-d (τ_c = 0.75, b = 0.7)	1.11	1.45	0.317	1.92	1.74	1.70	1.30	1.46
IMC (τ_c = 0.85)	1.11	1.50	0.333	1.94	1.65	2.21	1.35	1.51
ZN	1.36	1.55	0.387	2.59	1.81	4.46	1.16	2.79

ing example is used to illustrate how the proposed DS-d method works for processes with larger θ/τ ratios.

4.3. Example 3. Different Values of the θ/τ Ratio. Consider the process model

$$G_p(s) = G_d(s) = \frac{e^{-\theta s}}{s + 1} \tag{95}$$

In this example, two values of θ/τ ratio are considered: $\theta/\tau = 1$ and $\theta/\tau = 5$.

For $\theta/\tau = 1$, PI and PID controllers were designed using the DS-d, DS/IMC, and ZN methods. The controller characteristics are shown in Tables 4 and 5. The simulation results in Figures 5 and 6 indicate that the PI and PID responses for the DS-d and DS/IMC methods are very similar. The DS-d and DS/IMC controllers provide better performance than the ZN controllers.

For the process with $\theta/\tau = 5$, the DS-d, DS/IMC, and ZN methods were used to design PI and PID controllers. Luyben[2] has used this large-time-delay example to compare the IMC, Ciancone–Marlin (CM),[34] and ZN methods. Consequently, the CM PI and PID settings were also considered for comparison. The CM tuning rules were derived using an optimization procedure that incorporates considerations of performance, robustness, and saturation of the manipulated variable. The CM tuning rules are valid only for first-order plus time delay processes and are presented graphically.[34]

The controller characteristics are given in Tables 6 and 7. The responses to unit step changes in the set point and disturbance are shown in Figures 7 and 8. The DS-d and DS PI controllers give the fastest PI responses with small overshoots that are very similar. The responses of the ZN PI controller are very sluggish. The CM PI controller provides the best performance among all four PI controllers. For the PID controllers, the responses of the DS-d, IMC, and CM controllers are relatively close. The ZN PID controller gives very erratic

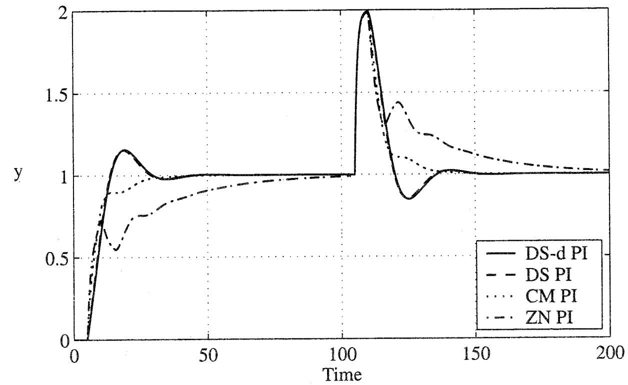

Figure 7. Simulation results of PI controllers for example 3 (θ/τ = 5).

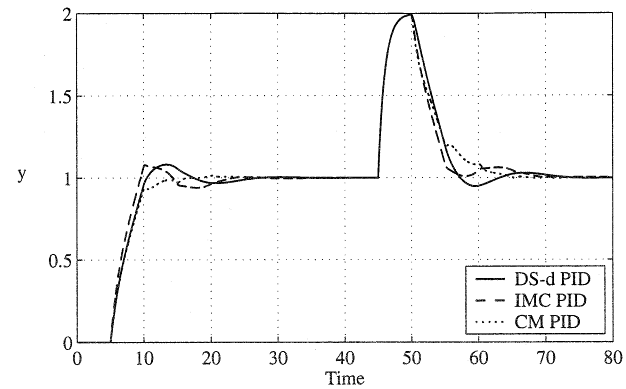

Figure 8. Simulation results of PID controllers for example 3 (θ/τ = 5).

Table 6. PI Controller Settings for Example 3 (θ/τ = 5)

				set point		disturbance	
tuning method	K_c	τ_I	M_S	IAE	TV	IAE	TV
DS-d (τ_c = 1.9)	0.11	0.87	1.86	10.9	1.27	10.8	1.37
DS (τ_c = 2.8)	0.13	1.00	1.86	10.6	1.24	10.5	1.36
CM	0.35	3.3	1.68	9.43	0.68	9.43	1.01
ZN	0.515	9.8	1.89	18.8	0.96	18.5	1.34

Table 7. PID Controller Settings for Example 3 (θ/τ = 5)

					set point		disturbance	
tuning method	K_c	τ_I	τ_D	M_S	IAE	TV	IAE	TV
DS-d (τ_c = 2.5)	0.4	2.86	0.313	1.86	7.69	0.939	7.39	1.18
IMC (τ_c = 4.5)	0.5	3.5	0.714	1.87	7.38	1.28	6.99	1.48
CM	0.4	3.0	1.2	1.92	7.60	1.20	7.52	1.57
ZN	0.66	5.9	1.48	42.2	9.48	5.94	9.26	9.64

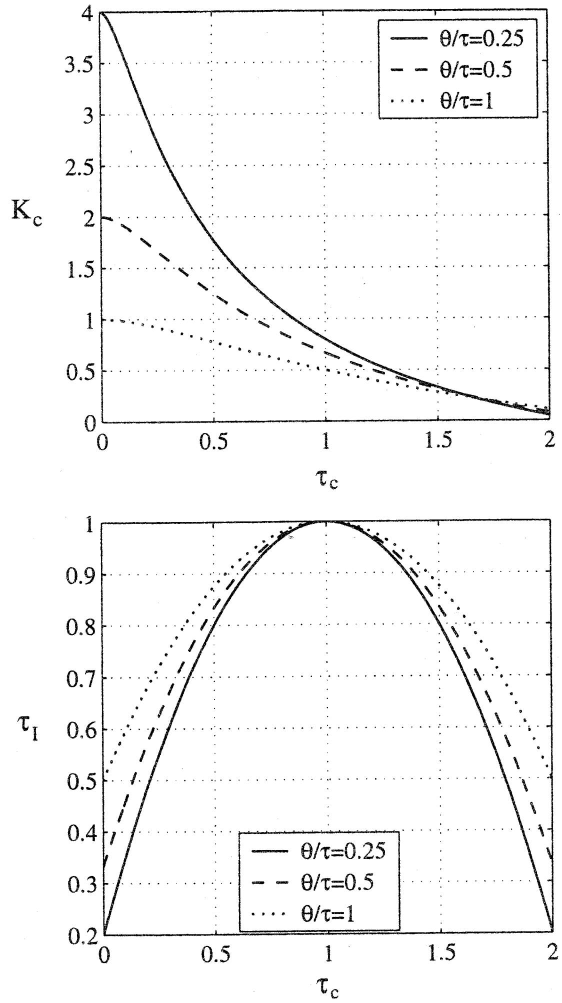

Figure 9. DS-d PI controller settings for different τ_c values (example 4).

responses that are not shown in the plot but can be found in Luyben.[2] Its M_S value in Table 7 is extremely large.

The results for these two θ/τ values illustrate that the DS-d method provides better disturbance rejection than the DS method for processes with small values of θ/τ. As θ/τ becomes larger, the controller settings and the closed-loop performance for the DS-d method become closer to those for the conventional DS method.

4.4. Example 4. Effect of τ_c. The DS-d method has a single tuning parameter, τ_c, that is directly related to the speed of the closed-loop response. In this example, the effect of τ_c is analyzed. Consider the general model

$$G_p(s) = G_d(s) = \frac{e^{-\theta s}}{s + 1} \tag{96}$$

For three different values of θ/τ (0.25, 0.5, 1), DS-d PI controller settings were calculated for different τ_c values.

Figure 9 shows the DS-d PI controller settings for different values of τ_c. As τ_c increases, K_c decreases, and the integral time, τ_I, increases if $\tau_c \leq 1$ but decreases if $\tau_c \geq 1$. The symmetry of τ_I around $\tau_c = \tau$ is confirmed by eq 36.

Similarly, for three different θ/τ values (0.25, 0.5, 0.75), the DS-d PID controller settings were calculated for different τ_c values and are shown in Figure 10. As τ_c increases, K_c decreases, while τ_I and τ_D first increase

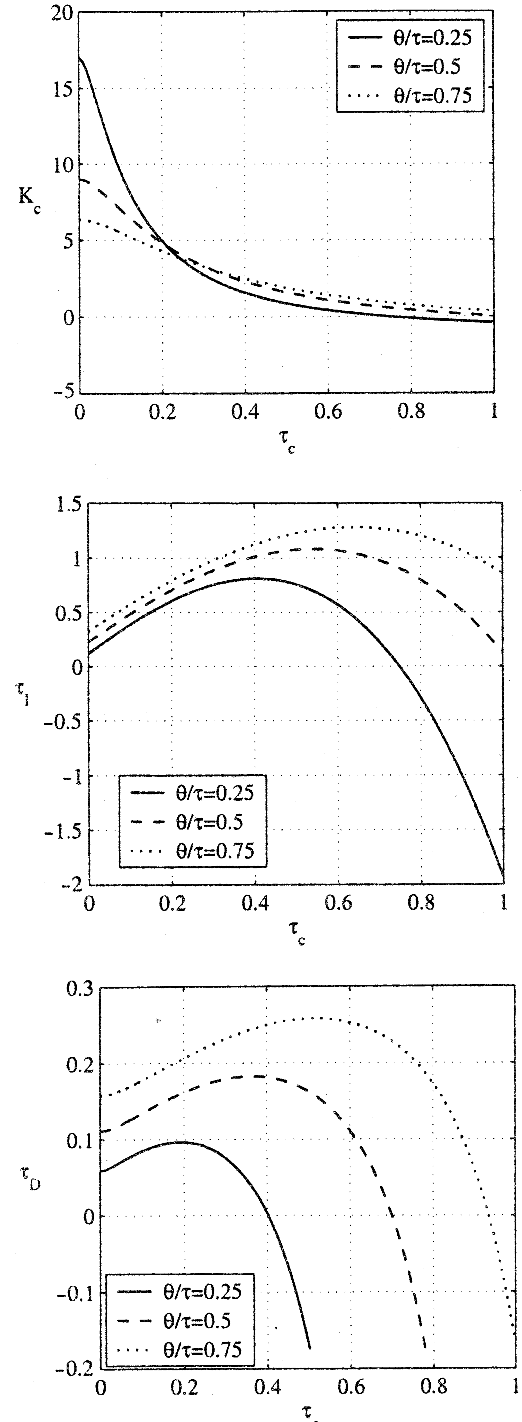

Figure 10. DS-d PID controller settings for different τ_c values (example 4).

and then decrease. If τ_c is very large, τ_I and τ_D become negative. Thus, for this example, the upper bounds on τ_c for positive K_c, τ_I, and τ_D values obtained from Figure 10 are

$$\tau_c \leq 0.4 \qquad \text{for } \theta/\tau = 0.25 \tag{97}$$

$$\tau_c \leq 0.7 \qquad \text{for } \theta/\tau = 0.5 \tag{98}$$

$$\tau_c \leq 0.935 \qquad \text{for } \theta/\tau = 0.75 \tag{99}$$

For $\theta/\tau = 0.25$, the DS-d PID controllers were designed for four values of τ_c (0.25, 0.3, 0.35, 0.4), and the

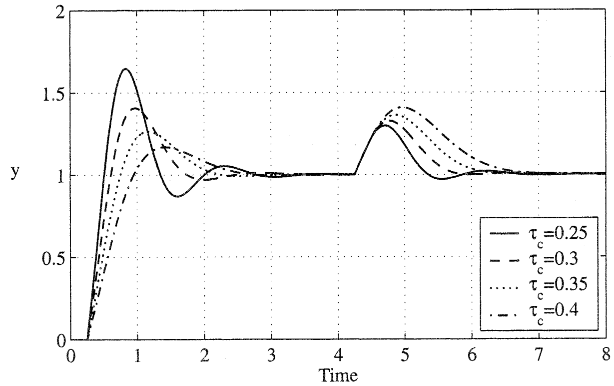

Figure 11. DS-d PID controllers for different τ_c values and $b = 1$ (example 4, $\theta/\tau = 0.25$).

Table 8. PID Controller Settings for Example 5 ($\theta/\tau = 0.25$)

tuning method	K_c	τ_I	τ_D	M_S
DS-d ($\tau_c = 0.26$, $b = 1$)	3.46	0.702	0.0887	1.89
DS-d ($\tau_c = 0.26$, $b = 0.5$)	3.46	0.702	0.0887	1.89
IMC ($\tau_c = 0.22$)	3.26	1.13	0.111	1.90
ZN	4.16	0.458	0.115	2.37

corresponding simulation results are shown in Figure 11. The simulation results demonstrate that the DS-d controllers for smaller τ_c values provide disturbance responses with smaller peaks and smaller IE values.

4.5. Example 5. Different Disturbance Time Constants. In this example, a process described by the same transfer function as in example 2

$$G_p(s) = \frac{e^{-0.25s}}{s + 1} \tag{100}$$

but disturbance transfer functions with different values of τ_d is considered

$$G_d(s) = \frac{e^{-0.25s}}{\tau_d s + 1} \tag{101}$$

PID controllers were designed using the DS-d, IMC, and ZN methods and assuming that $\tau_d = \tau$. The controller characteristics are shown in Table 8. The DS-d controller was modified for other values of τ_d by using eq 87. After substituting the transfer functions for the process and disturbance, the closed-loop transfer function for the DS-d PID controller in eq 87 becomes

$$\left(\frac{y}{d}\right)^* \approx \frac{1.125s(s + 1)e^{-0.25s}}{(\tau_d s + 1)(\tau_c s + 1)^3} \tag{102}$$

Thus, when τ_d/τ increases, the disturbance response of the DS-d PID controller has a smaller peak value and is more sluggish.

Because τ_d does not affect the set-point response, only the disturbance responses were simulated. The simulation results for four τ_d values (0.25, 0.5, 1, 2) shown in Figure 12 confirm that DS-d controllers provide disturbance responses with smaller peak values but longer settling times as τ_d/τ increases. For small τ_d/τ values, the disturbance performance of the IMC controller is better than that of the DS-d controller.

4.6. Example 6. Consider a second-order plus time delay system described by Seborg et al.[15]

$$G_p(s) = G_d(s) = \frac{2e^{-s}}{(10s + 1)(5s + 1)} \tag{103}$$

The DS-d, DS, and ZN methods were used to design PID controllers, and the resulting controller settings are shown in Table 9.

The simulation results for a unit step change in the set point at $t = 0$ min and a unit step disturbance at $t = 50$ min are presented in Figure 13. The disturbance response for the DS-d PID controller is fast and has a small peak value, but the set-point response has an overshoot. By using a set-point weighting factor of $b = 0.5$, the overshoot is eliminated. The set-point and disturbance responses for the DS PID controller are quite sluggish, whereas the ZN PID controller provides very oscillatory responses. Again, the IAE and TV values in Table 9 confirm that the DS-d controller provides superior performance without using excessive control efforts.

4.7. Example 7. Distillation Column Model. A distillation column that separates a small amount of a low-boiling material from the final product was considered by Chien and Fruehauf.[1] The bottom level of the distillation column is controlled by adjusting the steam flow rate. The process model for the level control system is an integrator with a time delay

$$G_p(s) = G_d(s) = \frac{0.2e^{-7.4s}}{s} \tag{104}$$

The DS controller obtained from eq 12 has the form

$$G_c(s) = \frac{1}{K(\tau_c + \theta)} \tag{105}$$

Because the DS method results in a proportional-only controller for the integrating process, only the DS-d tuning methods was used to design PI controllers. Note that the DS-d and IMC methods provide identical PI tuning rules for this type of process model. The resulting controller settings from the DS-d method are shown in Table 10. The IMC PI controller settings with $\tau_c = 8$ used by Chien and Fruehauf[1] are included in the table. Furthermore, for integrator plus time delay processes, Tyreus and Luyben[35] have developed a design method that yields the best PI settings attainable for a specified degree of closed-loop damping. Their tuning rule can be expressed in terms of the ultimate gain K_u and ultimate frequency P_u as $K_c = K_u/3.22$ and $\tau_I = 2.2P_u$. Thus, it is a modified version of ZN tuning. The TL PI controller settings[35] are included in Table 10 for comparison.

The simulation results for a unit step change in the set point (at $t = 0$) and a 0.5 step disturbance (at $t = 150$ min) are shown in Figure 14. The disturbance performance of the DS-d controller is good, but the set-point response has a large overshoot that can be eliminated by setting $b = 0.5$. The IMC PI controller designed by Chien and Fruehauf[1] provides a faster disturbance response than the DS-d method, but the set-point response is too aggressive, as confirmed by the large M_S and TV values in Table 10. The use of set-point weighting can reduce the large set-point overshoot for the IMC controller, but it would not affect the oscillatory nature of the response. The large M_S value

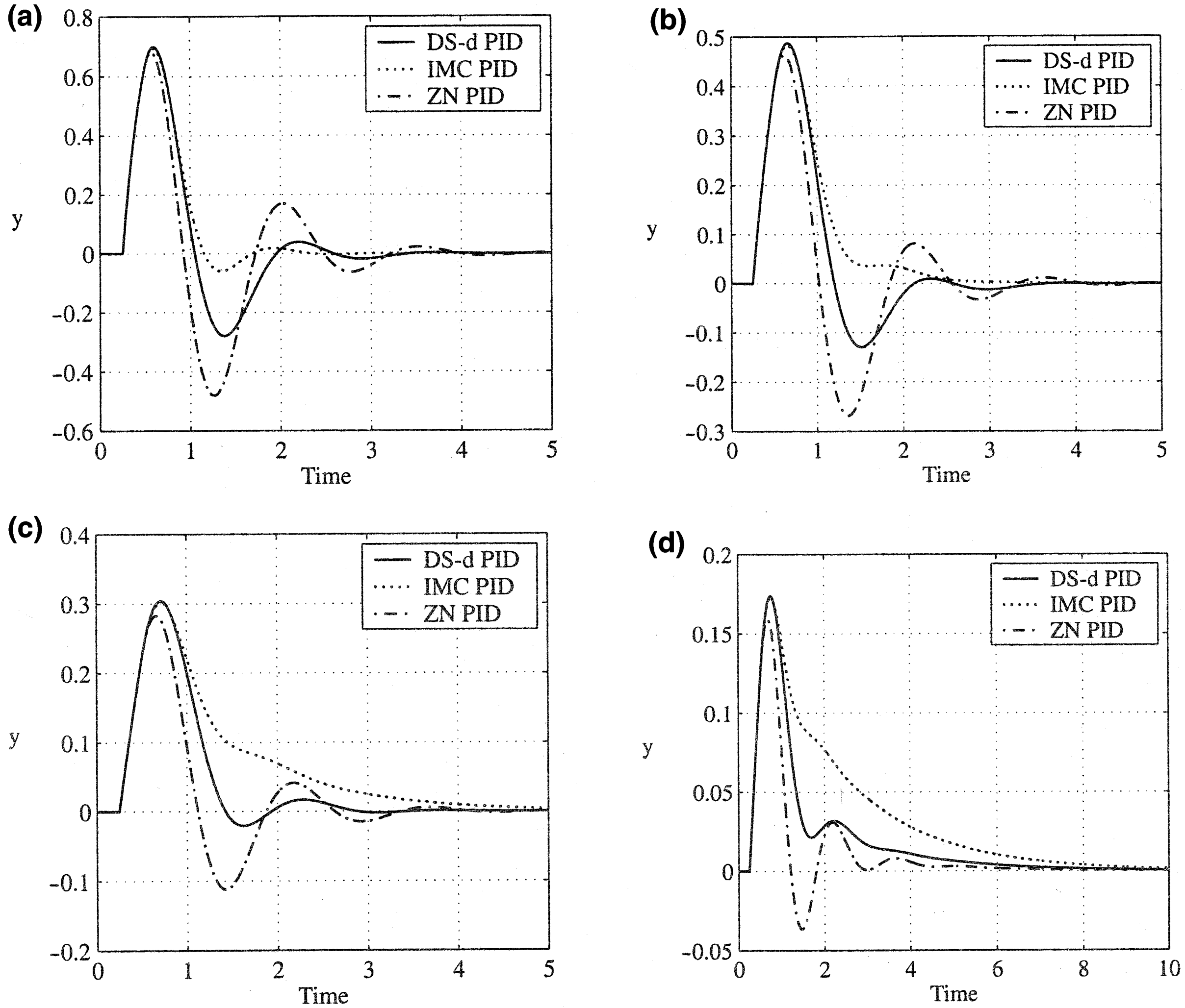

Figure 12. Disturbance responses for PID control and different τ_d values (example 5, $\theta/\tau = 0.25$). (a) $\tau_d = 0.25$. (b) $\tau_d = 0.5$. (c) $\tau_d = 1$. (d) $\tau_d = 2$.

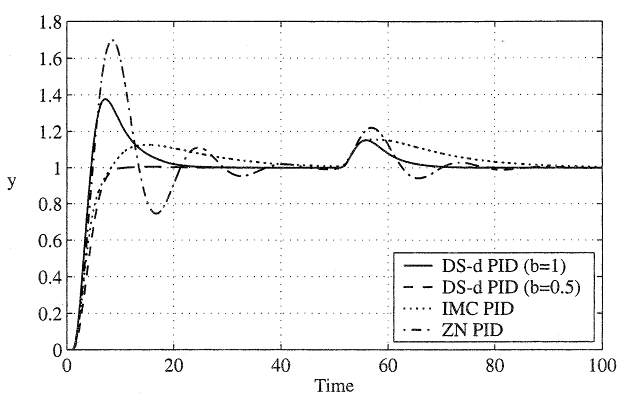

Figure 13. Simulation results for example 6.

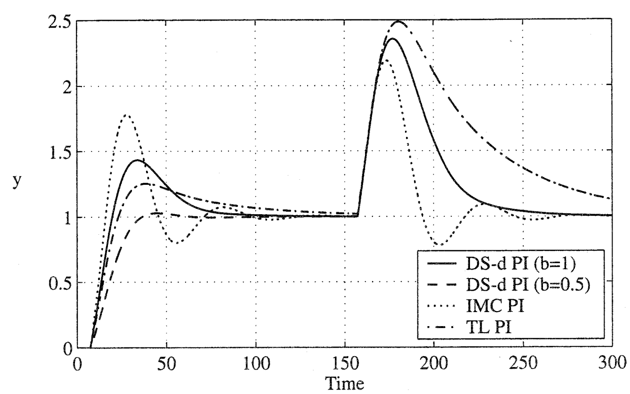

Figure 14. Simulation results for example 7.

Table 9. PID Controller Settings for Example 6

					set point		disturbance	
tuning method	K_c	τ_I	τ_D	M_S	IAE	TV	IAE	TV
DS-d ($\tau_c = 2.4$, $b = 1$)	6.3	7.60	2.10	1.87	5.59	13.3	1.19	2.10
DS-d ($\tau_c = 2.4$, $b = 0.5$)	6.3	7.60	2.10	1.87	4.58	6.78	1.19	2.10
DS ($\tau_c = 0.5$)	5	15	3.33	1.92	6.25	11.7	3.03	2.34
ZN	4.72	5.83	1.46	2.27	8.41	12.9	1.74	2.77

Table 10. PI Controller Settings for Example 7

				set point		disturbance	
tuning method	K_c	τ_I	M_S	IAE	TV	IAE	TV
DS-d ($\tau_c = 15$, $b = 1$)	0.373	37.4	1.94	27.1	0.675	50.1	0.932
DS-d ($\tau_c = 15$, $b = 0.5$)	0.373	37.4	1.94	19.6	0.354	50.1	0.932
IMC ($\tau_c = 8$)	0.49	23	3.06	30.9	1.63	30.9	1.58
TL	0.33	64.7	1.67	28.4	0.455	93.2	0.742

indicates that the τ_c value used by Chien and Fruehauf[1] is so small that the resulting system has poor robustness. The responses of the TL PI controller are very sluggish because of the large τ_I value.

4.8. Example 8. Level Control Problem. Consider a level control problem given by Seborg et al.[15] The liquid level in a reboiler of a steam-heated distillation column is to be controlled by adjusting the control valve

Table 11. PID Controller Settings for Example 8

					set point		disturbance	
tuning method	K_c	τ_I	τ_D	M_S	IAE	TV	IAE	TV
DS-d ($\tau_c = 1.6$, $b = 1$)	−1.25	5.3	1.45	1.94	3.42	3.39	4.33	2.86
DS-d ($\tau_c = 1.6$, $b = 0.5$)	−1.25	5.3	1.45	1.94	2.82	1.62	4.33	2.86
IMC ($\tau_c = 1.25$)	−1.22	6.0	1.50	1.96	3.48	3.29	4.99	2.83
ZN	−0.752	3.84	0.961	2.76	7.17	2.86	9.31	3.91

on the steam line. The process transfer function is given by

$$G_p(s) = G_d(s) = \frac{-1.6(-0.5s + 1)}{s(3s + 1)} \tag{106}$$

The PID settings obtained from the DS-d, IMC, and ZN methods are shown in Table 11.

The simulation results for a unit step change in the set point at $t = 0$ min and a unit step disturbance at $t = 50$ min are presented in Figure 15. The disturbance response for the DS-d PID controller is good, but the set-point response has an overshoot. With a set-point weighting factor set at $b = 0.5$, the DS-d controller provides a fast set-point response without overshoot. The performance of the IMC PID controller is close to that of the DS-d PID controller, which is confirmed by the similar PID settings. The ZN controller produces very oscillatory responses.

4.9. Example 9. Fourth-Order Process. Consider a fourth-order process described by[29,30]

$$G_p(s) = G_d(s) = \frac{1}{(s + 1)(0.2s + 1)(0.04s + 1)(0.008s + 1)} \tag{107}$$

For high-order processes, the DS-d, DS, and IMC design methods do not yield PI/PID controllers directly. Thus, the model order must be reduced, or the resulting controller must be approximated by a PI/PID controller. Skogestad[30] has proposed a simple method of approximating high-order models with low-order models. He also derived modified IMC rules, which were named "simple control" or "Skogestad IMC" (SIMC). For the first-order model in eq 30, the PI controller obtained from SIMC has the following parameters

$$K_c = \frac{\tau}{K(\tau_c + \theta)}, \quad \tau_I = \min\{\tau, 4(\tau_c + \theta)\} \tag{108}$$

For the second-order model in eq 67 with $\tau_1 > \tau_2$, the SIMC method provides the following rules for PID controllers with the series structure

$$K_c = \frac{\tau_1}{K(\tau_c + \theta)}, \quad \tau_I = \min\{\tau_1, 4(\tau_c + \theta)\}, \quad \tau_D = \tau_2 \tag{109}$$

Using Skogestad's approximation method, the fourth-order process in eq 107 can be approximated as a first-order plus time delay model

$$G_1(s) = \frac{e^{-0.148s}}{1.1s + 1} \tag{110}$$

or as a second-order plus time delay model

$$G_2(s) = \frac{e^{-0.028s}}{(s + 1)(0.22s + 1)} \tag{111}$$

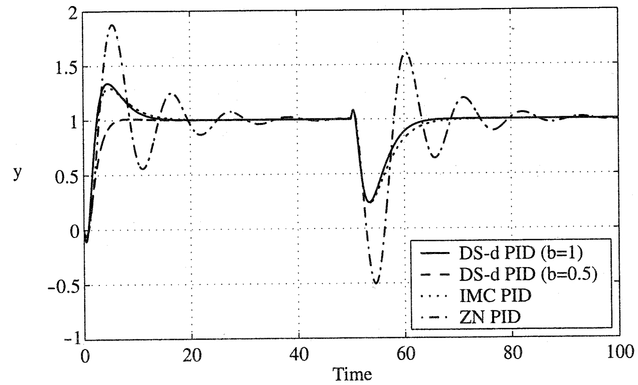

Figure 15. Simulation results for example 8.

Table 12. PI Controller Settings for Example 9

				set point		disturbance	
tuning method	K_c	τ_I	M_S	IAE	TV	IAE	TV
DS-d ($\tau_c = 0.4$, $b = 1$)	2.94	0.707	1.61	0.553	3.66	0.721	4.56
DS-d ($\tau_c = 0.4$, $b = 0.5$)	2.94	0.707	1.61	0.594	2.08	0.721	4.56
DS ($\tau_c = 0.148$)	3.72	1.1	1.59	0.450	4.45	0.874	4.22
SIMC ($\tau_c = 0.148$)	3.72	1.1	1.59	0.450	4.45	0.874	4.22
TL	9.46	1.24	2.72	0.498	25.9	0.387	8.67

Table 13. PID Controller Settings for Example 9

					set point		disturbance	
tuning method	K_c	τ_I	τ_D	M_S	IAE	TV	IAE	TV
DS-d ($\tau_c = 0.135$, $b = 1$)	22.4	0.415	0.106	1.58	0.276	38.0	0.056	5.56
DS-d ($\tau_c = 0.135$, $b = 0.5$)	22.4	0.415	0.106	1.58	0.231	18.3	0.056	5.56
IMC ($\tau_c = 0.025$)	22.6	1.2	0.167	1.58	0.311	48.3	0.160	6.57
SIMC ($\tau_c = 0.028$)	21.8	1.22	0.18	1.58	0.333	48.7	0.168	6.85
ZN	18.1	0.281	0.07	2.38	0.423	52.0	0.070	9.03

Both models provide accurate approximations, but the second-order model is more accurate.

PI controllers were designed using the approximate first-order model and the DS-d, DS, and SIMC methods. Values of $\tau_c = 0.4$ and 0.148 were selected for the DS-d and DS methods, respectively, so that their M_S values were very close to the 1.59 value for the SIMC controller reported by Skogestad.[30] For $\tau_c = 0.148$, the same PI controller settings were calculated using the DS and SIMC methods. Also, the TL method,[35] which was discussed in example 7, was used to design a PI controller for the process. The PI settings are shown in Table 12.

PID controllers were designed using the approximate second-order model and the DS-d, IMC, and SIMC methods. The τ_c values for these methods were selected to give M_S values close to the 1.58 value of Table 13. Because the tuning rules for the SIMC method are for the series PID structure, these PID settings were converted to the parallel structure, which was then used for the simulation. Also, the ZN PID settings were calculated for the fourth-order model. The PID controller characteristics are shown in Table 13.

The simulation responses for a unit step change in the set point at $t = 0$ min and a step disturbance ($d = 3$) at $t = 5$ min are given in Figures 16 and 17 for PI and PID controllers, respectively. The values of M_S, IAE, and TV for all of these controllers are presented in Tables 12 and 13. Figure 16 indicates that the disturbance response of the DS-d PI controller is better and faster than the responses of the DS and SIMC controllers. The IAE values for the TL PI controller are smaller than those of the other PI controllers, but its responses are oscillatory and its M_S and TV values are large. A

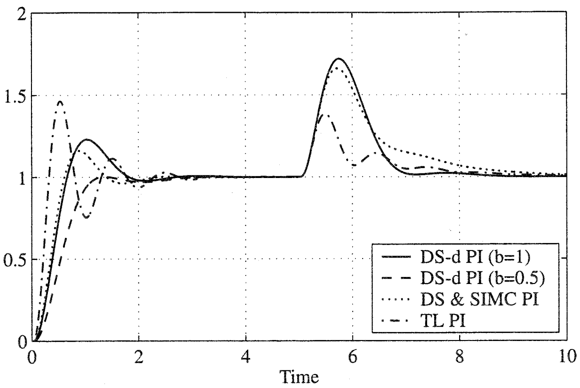

Figure 16. Simulation results of PI controllers for example 9.

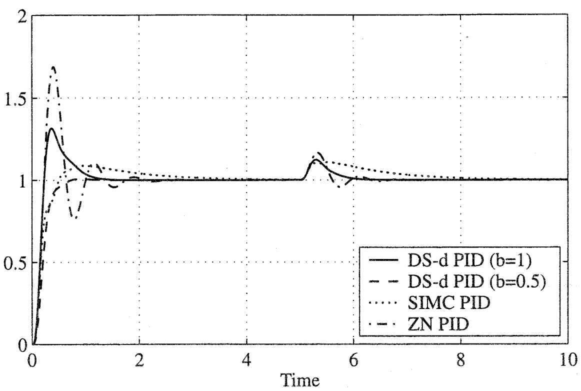

Figure 17. Simulation results of PID controllers for example 9. (The responses of the IMC PID controller are very similar to those of the SIMC PID controller.)

comparison of the PID controllers in Figure 17 indicates that the DS-d PID controller provides better and faster disturbance responses than the other PID controllers. The large overshoot for the set-point response of the DS-d PID controller can be eliminated by setting $b = 0.5$. The superior performance of the DS-d PID controller is also confirmed by the IAE and TV values in Table 13. Furthermore, a significant improvement was obtained using PID control instead of PI control, because this process is a dominant second-order process.

An alternative approach for high-order systems is to design the DS-d controller using eq 25 and then reduce the resulting high-order controller using a series expansion[22] or a frequency domain approximation.[24–26] Because this approach is more complicated than Skogestad's model reduction approach, it was not applied here.

5. Conclusions

A new direct synthesis method for controller design based on disturbance rejection (DS-d), rather than set-point tracking, has been developed. By specifying the desired closed-loop transfer function properly, PI/PID controllers can be synthesized for widely used process models such as first-order and second-order plus time delay models and integrator plus time delay models. For higher-order models, PID controllers can be derived by approximating the high-order model with a low-order model or by approximating the high-order controller using either a series expansion or a frequency domain approximation.

In the proposed DS-d design method, the closed-loop time constant τ_c is the only design parameter, and it has a straightforward relation to the disturbance rejection characteristics. Thus, the proposed design procedure is simple and easy to implement. Although the PI/PID controllers are designed for disturbance rejection, the set-point responses are usually satisfactory and can be independently tuned via a standard set-point weighting factor or a set-point filter constant. The set-point tuning does not affect the disturbance response.

Nine simulation examples have been used to compare alternative design methods and to illustrate the effect of τ_c. The simulation results demonstrate that the DS-d method provides better disturbance performance than standard DS and IMC methods and that satisfactory responses to set-point changes can be obtained by simply tuning the set-point weighting factor b. The DS-d method furnishes a convenient and flexible design method that provides good performance in terms of disturbance rejection and set-point tracking.

Acknowledgment

The authors acknowledge the UCSB Process Control Consortium for financial support and Professor Sigurd Skogestad for his helpful comments and suggestions.

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Received for review September 7, 2001
Revised manuscript received January 22, 2002
Accepted July 1, 2002

IE010756M