Networked and Distributed Parameter Systems
(Some) New Directions, Opportunities & Challenges

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Note: These slides represent a synthesis of two semi-plenary talks given at the American Control Conference (ACC) and the European Control Conference (ECC) respectively in June of 2014

Complexity and Performance in Large-Scale and Distributed Systems

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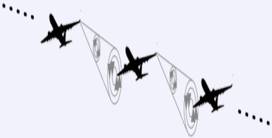
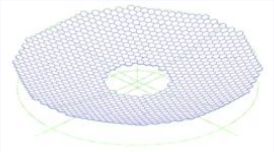
EUROPEAN CONTROL CONFERENCE

Networked/Cooperative/Distributed Control

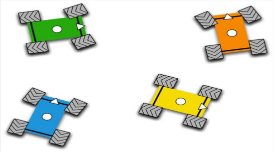


aircraft formation flight

formation flight in nature



large telescope arrays



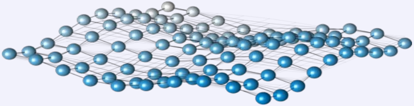
robotic networks

flocks & swarms

automated highways

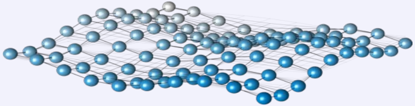
- An area rich in deep and interesting problems
- Rapidly evolving

Networked vs. Distributed Parameter Systems

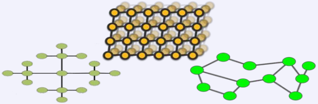
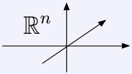
Spatially Distributed Systems	
Networked/Cooperative/Distributed Control	Distributed Parameter Systems
	

Networked vs. Distributed Parameter Systems

SPATIALLY DISTRIBUTED SYSTEMS

Networked/Cooperative/Distributed Control	Distributed Parameter Systems
	

CORRESPONDENCES (Physics/Numerical Analysis perspective)

discrete space described by graph structure	*continuum space*
	 

differential equations over large graphs

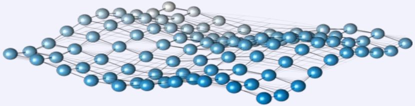
Numerical Methods ⟵

⟶ Continuum Models

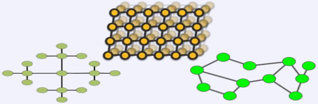
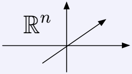
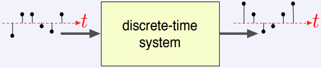
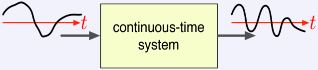
Partial Differential Equations

Networked vs. Distributed Parameter Systems

Spatially Distributed Systems

Networked/Cooperative/Distributed Control	Distributed Parameter Systems
	

Analogy with temporal systems (Systems & Controls perspective)

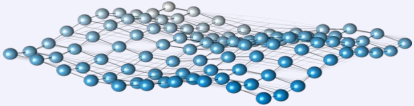
discrete space described by graph structure	*continuum space*
	 
	

UNIFYING PERSPECTIVE: *Spatio-temporal* systems over discrete or continuum space

- Signals over continuous and/or discrete time and space
- Investigate systems properties (e.g. system norms & responses)

Outline

SPATIALLY DISTRIBUTED SYSTEMS

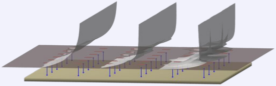
Networked/Cooperative/Distributed Control	Distributed Parameter Systems
	

LOOK AT SPECIFIC PROBLEMS

— Vehicular Strings and Consensus

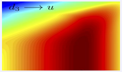
— Structured Control Design

— Flow Turbulence & Control



— Spatio-temporal

Impulse Responses



Frequency Responses

Outline

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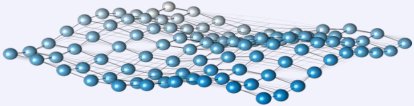
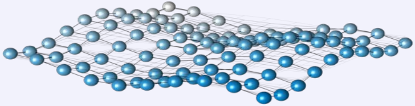
	

SOME COMMON THEMES EMERGE

- *The use of system norms and responses*
- *Large-scale (even linear) systems exhibit some surprising phenomena*
- *Large-scale & Regular Networks* $\longrightarrow$ *Asymptotic statements* *(in system size)*
- *Network topology imposes asymptotic "hard performance limits"*

Vehicular Strings (Platoons)

Spatially Distributed Systems

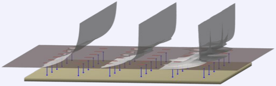
Networked/Cooperative/Distributed Control	Distributed Parameter Systems
	

Look at specific problems

— Vehicular Strings and Consensus

— Structured Control Design

— Flow Turbulence & Control



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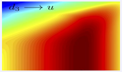
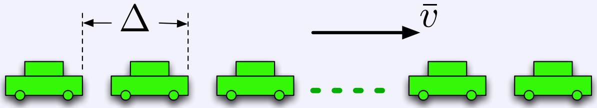
Impulse Responses



Frequency Responses

Vehicular Platoons

Automated control of each vehicle, tight spacing at highway speeds



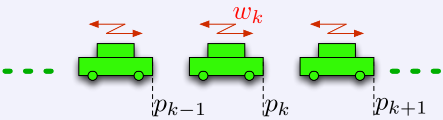
- Is it enough to look at neighbors? Should information be broadcast to all?
- How does performance scale with size?
- Are there any fundamental limitations?

A fundamentally difficult problem (scales badly with size)

due to the network topology

Vehicular Platoons (setting)

$$\ddot{p}_k \;=\; \underset{\text{control}}{\underset{\uparrow}{u_k}} \;+\; \underset{\text{disturbance}}{\underset{\uparrow}{w_k}}$$



- *Desired trajectory:* $\bar{p}_k \;:=\; \bar{v}t \;+\; k\Delta$ *constant velocity*
- Deviations:

$$\tilde{p}_k \;:=\; p_k \;-\; \bar{p}_k, \qquad \tilde{v}_k \;:=\; \dot{p}_k - \bar{v}$$

- Controls:

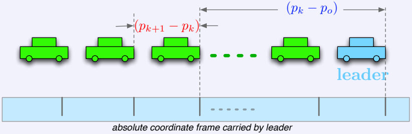
$$u \;=\; K\tilde{p} \;+\; F\tilde{v}$$

- Closed loop:

$$\frac{d}{dt}\begin{bmatrix} \dot{\tilde{p}} \\ \dot{\tilde{v}} \end{bmatrix} = \begin{bmatrix} 0 & I \\ K & F \end{bmatrix}\begin{bmatrix} \tilde{p} \\ \tilde{v} \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} w$$

K, F: matrix feedback gains (look like "Laplacians" $\approx$ 2nd order consensus)

Relative vs. Absolute Feedback



position feedback → $K\,\tilde{p}$; velocity feedback → $F\,\tilde{v}$

$$u \;=\; K\,\tilde{p} \;+\; F\,\tilde{v}$$

$$\begin{aligned} u_k \;=\;& K_+\,(p_{k+1} - p_k - \Delta) \;+\; K_-\,(p_k - p_{k-1} - \Delta) \;+ \\ & F_+\,(v_{k+1} - v_k) \;+\; F_-\,(v_k - v_{k-1}) \;+ \\ & K_o\,(p_k \;-\; (vt + \Delta k)) \;+ \\ & F_o\,(v_k \;-\; \bar{v}) \end{aligned}$$

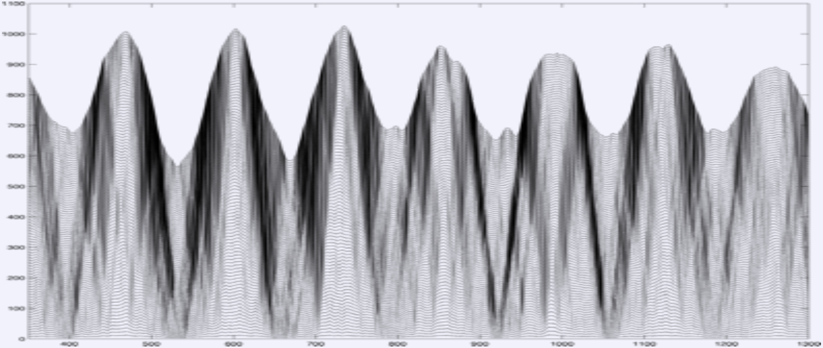
- **Relative Measurements:**
  - Requires ranging devices

$\Leftrightarrow$ $\text{row\_sums}(K) = 0$, $\text{row\_sums}(F) = 0$

- **Absolute Measurements:**
  - Position: Requires knowing position relative to leader
  - Velocity: Requires measurement of own velocity

Disorder Phenomenon in Platoons (w. only relative meas.)

Globally stable formation, but exhibits “accordion-like” large-scale modes



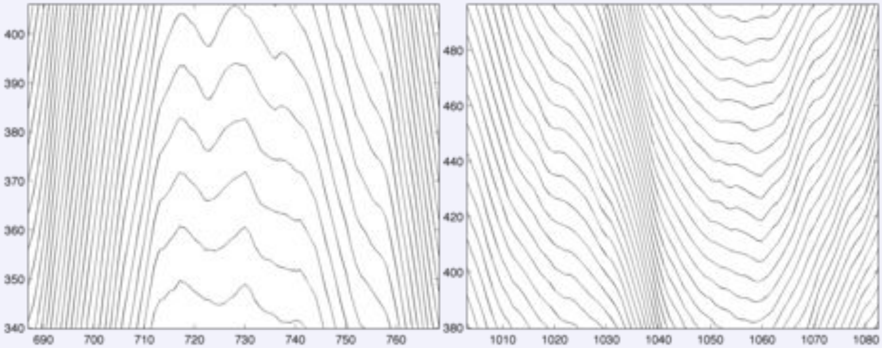
Time trajectories of vehicles’ positions relative to leader (bird’s-eye view)

100 vehicles

-A large formation in a thunderstorm

Disorder Phenomenon in Platoons (w. only relative meas.)

Zoomed in (small-scale) behavior

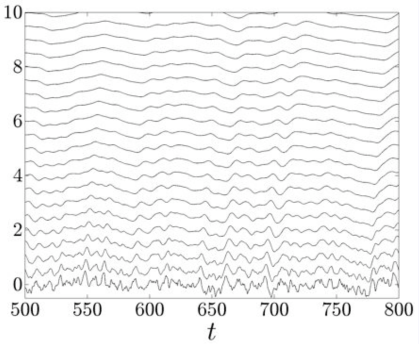


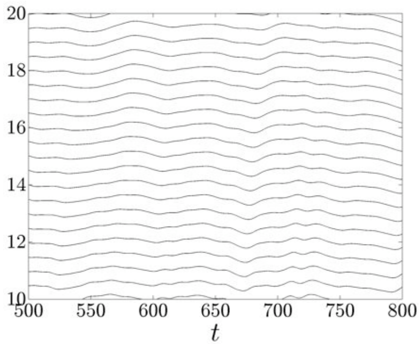
- Seems well regulated. No collisions.
- Unrelated to "string instability". A different phenomenon.

Disorder Phenomenon in Platoons (w. only relative meas.)

String instability?

Let disturbances enter only at lead vehicle



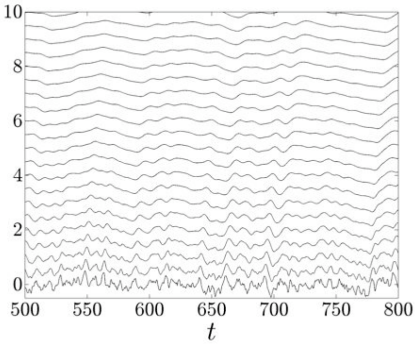


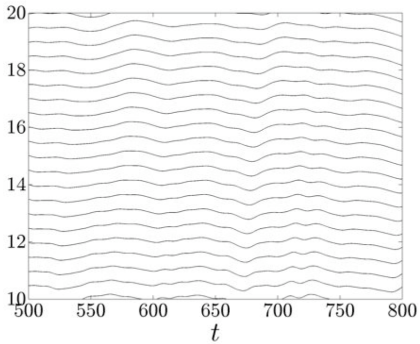
Unrelated to string instability!

Disorder Phenomenon in Platoons (w. only relative meas.)

String instability?

Let disturbances enter only at lead vehicle

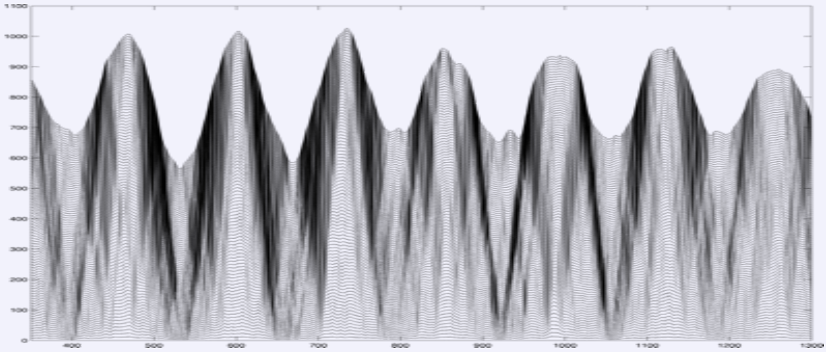




- temporally high frequency disturbances well regulated
- temporally low frequency disturbances penetrate further into formation

Disorder Phenomenon in Platoons (w. only relative meas.)

Globally stable formation, but exhibits "accordion-like" large-scale modes



This motion dominated by

- Temporally slow modes
- Large spatial scales

} "Global" modes

Vehicular Platoons (Optimal LQR)

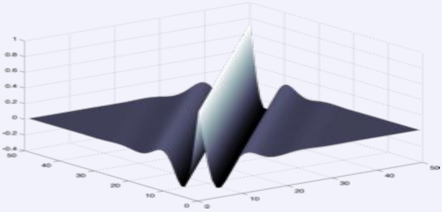
- Is this due to bad design, or is it inherent to this problem?
- Note: Also occurs in LQR controllers that yield “localized” feedbacks
  - Original formulations:
    - Athans & Levine ’66
    - Melzer & Kuo ’70
  - Reexamined as $N \longrightarrow \infty$
    - *Jovanovic & Bamieh, TAC ’05*

Vehicular Platoons (Optimal LQR)

Centralized LQR design *(Melzer & Kuo '70, Athans & Levine '66)*

$$
\begin{aligned}
\begin{bmatrix} \dot{\tilde{x}} \\ \dot{\tilde{v}} \end{bmatrix} &= \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{x} \\ \tilde{v} \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} w, \\
J &= \int_0^\infty \sum_k \left(q_1 \left(\tilde{x}_k - \tilde{x}_{k-1} \right)^2 + q_2 \, \tilde{v}_k^2 + u_k^2 \right)
\end{aligned}
$$

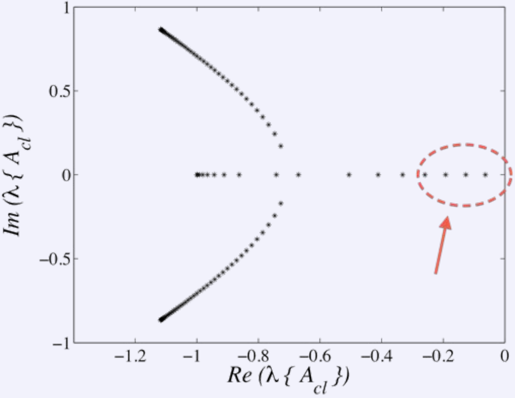
Feedback gains are "localized":



Inherent Localization: *Bamieh et. al, TAC '02, Motee et. al. '07*

Vehicular Platoons (Optimal LQR)

Closed loop eigenvalues of optimal LQR feedback



- neutrally stable "mean mode" at $\lambda_1 = 0$ does not effect stability
- however, it attracts an *unbounded number* of eigenvalues as $N \to \infty$

Not string instability! Long wavelength modes are problematic

This system's modes: long spatial wavelength $\leftrightarrow$ slow temporal scale

Vehicular Platoons LQR (infinite limit)

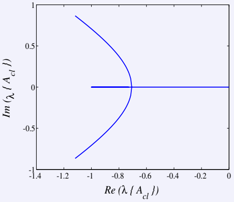
"Infinity is a convenient approximation to a large number" -Anonymous

Infinite platoon $\longrightarrow$ Spatially invariant $\longrightarrow$ Transform analysis

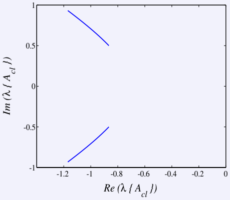
$$\begin{aligned}\begin{bmatrix}\dot{\tilde{p}}_k\\ \dot{\tilde{v}}_k\end{bmatrix} &= \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}\tilde{p}_k\\ \tilde{v}_k\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix} u_k, \quad k \in \mathbb{Z}\\ J &= \int_0^\infty \sum_{k\in\mathbb{Z}} \left(q\,\tilde{p}_k^2 \;+\; (\tilde{p}_k - \tilde{p}_{k-1})^2 \;+\; \tilde{v}_k^2 \;+\; u_k^2 \right) dt\end{aligned}$$

Resulting closed loop spectra

$q \;=\; 0$:



$q \;=\; 1$:

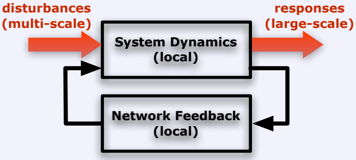


However: adding absolute penalty $q\tilde{p}_k^2$ yields non-local optimal feedback

Jovanovic & Bamieh, TAC '05

Disorder and Feedback “Granularity”

- Disturbances are spatially white (contain all spatial wavelengths)



- Intuition:
  - *Local feedback* can only suppress *short-scale disturbances*
  - Local feedback ineffective against large-scale (& slow) disturbances
  - Looks like *global feedback* is needed for *global regulation*

Disorder and Feedback "Granularity"

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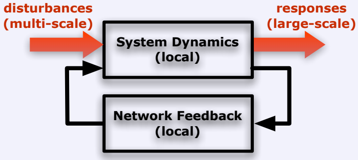

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  - Local feedback ineffective against large-scale (& slow) disturbances
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Surprise: In higher spatial dimensions:

Local feedback CAN suppress large-scale disturbances

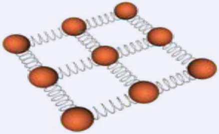
cf. Harmonic Solids

Statistical Mechanics of Harmonic Solids

Harmonic solid: A d-dimensional lattice of masses and springs

Q: Can *short range interaction* lead to *long range order*?

- "short range interaction" $\longleftrightarrow$ local feedback
- "long range order" $\longleftrightarrow$ tightness of formation



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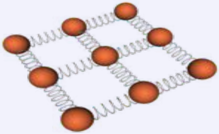

Studied using *long range correlations*

- for $d = 1, 2$ short range interactions $\Rightarrow$ no long range order
- for $d \geq 3$ long range order possible!
- i.e., solids can only exist in $d \geq 3$

Statistical Mechanics of Harmonic Solids

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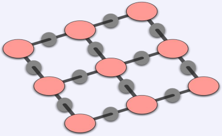

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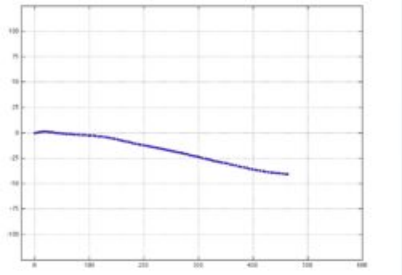
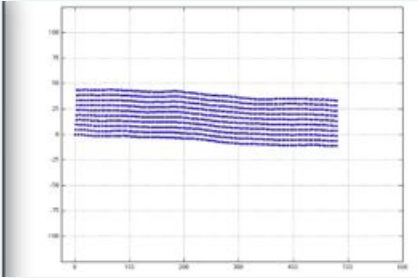
Similar dimentional-dependencies occur in networked control systems?

Comparison between 1D and (semi)2D cases





Related Concepts

- Optimal Performance of Distributed Estimation

(Barooah, Hespanha)

- Effective Resistance in a Resistor Network

(Lovisari, Garin, Zampieri, Carli)

- Global Mean First Passage Time of Simple Random Walk
- Wiener Index for Molecules

Common mathematical problem: calculate sums like (cont. time)

$$\sum_{n \neq 1} \frac{1}{\lambda_n}$$

λ_n: eigenvalues of a graph Laplacian

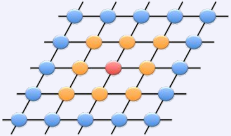
Setting:

- $N = M^d$ vehicles arranged in d-dimensional torus $\mathbb{Z}_M^d$
- Desired trajectory: $\bar{p}_k := vt + k\Delta$ constant speed & heading

Structural Constraints

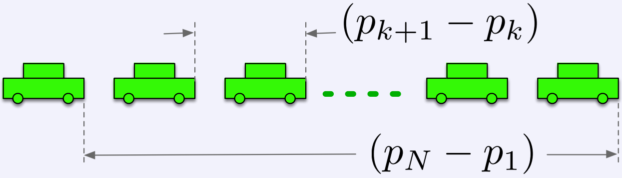
- *Spatial Invariance:*
  State-feedbacks K and F are spatial-convolution operators
- *Locality:* $K_{(k_1,\ldots,k_d)} = 0$, if for any $i \in \{1,\ldots,d\}$, $|k_i| > q$

feedback from local neighbors only



Performance Measures

- Two measures of "disorder"
  - **Microscopic**: *local position deviation* $\mathrm{var}\,(p_{k+1} - p_k - \Delta)$



  - **Macroscopic**: *long range deviation* $\mathrm{var}\,(p_N - p_1 - \Delta N)$

    or $\mathrm{var}\left(\tilde{p}_k - \frac{1}{N}\sum_l \tilde{p}_l\right)$

- All above obtained *asymptotically* (as $N \to \infty$) from H^2 norm calculations

Asymptotic Performance *Lower Bounds*

Tori networks, network size $= N$, spatial dimension $= d$, control effort $= \mathcal{E}\{u_k^2\} \leq U$

Feedback Type	Microscopic Disorder	Macroscopic Disorder
1st order consensus	$\frac{1}{U}$	$\frac{1}{U}\begin{cases} N & d=1 \\ \log(N) & d=2 \\ 1 & d\geq 3 \end{cases}$
absolute position & absolute velocity	$\frac{1}{U}$	$\frac{1}{U}$
relative position & absolute velocity	$\frac{1}{U}$	$\frac{1}{U}\begin{cases} N & d=1 \\ \log(N) & d=2 \\ 1 & d\geq 3 \end{cases}$
relative position & relative velocity	$\frac{1}{U^2}\begin{cases} N & d=1 \\ \log(N) & d=2 \\ 1 & d\geq 3 \end{cases}$	$\frac{1}{U^2}\begin{cases} N^3 & d=1 \\ N & d=2 \\ N^{1/3} & d=3 \\ \log(N) & d=4 \\ 1 & d\geq 5 \end{cases}$

"Coherence in Large-Scale Networks: Dimension-Dependent Limitations of Local Feedback"

BB, Jovanovic, Mitra, Patterson TAC, 2012

Implications for Vehicular Platoons

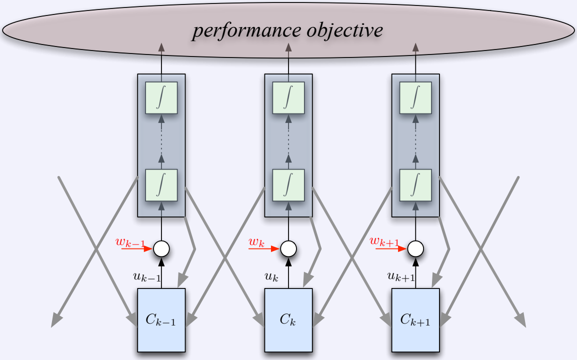
Feedback Type	Microscopic Disorder	Macroscopic Disorder
1st order consensus	$\frac{1}{U}$	$\frac{1}{U}\begin{cases} N & d=1 \\ \log(N) & d=2 \\ 1 & d\geq 3 \end{cases}$
absolute position & absolute velocity	$\frac{1}{U}$	$\frac{1}{U}$
relative position & absolute velocity	$\frac{1}{U}$	$\frac{1}{U}\begin{cases} N & d=1 \\ \log(N) & d=2 \\ 1 & d\geq 3 \end{cases}$
relative position & relative velocity	$\frac{1}{U^2}\begin{cases} N & d=1 \\ \log(N) & d=2 \\ 1 & d\geq 3 \end{cases}$	$\frac{1}{U^2}\begin{cases} N^3 & d=1 \\ N & d=2 \\ N^{1/3} & d=3 \\ \log(N) & d=4 \\ 1 & d\geq 5 \end{cases}$

Using only local feedback:

cannot have 1 dimensional, large and yet coherent formations!

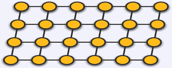
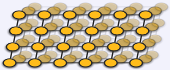
Role of Node Dynamics

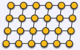
- Each node a chain of n integrators
- Controllers use local *static* state feedback



- Critical dimension needed for global coherence $= 2n + 1$
- Tradeoff between *network connectivity* and *node memory*

Spatial Dimension as Proxy for Network Connectivity

Convergence Time $1/\lambda_2$	N^2	N	$N^{2/3}$	$N^{2/d}$
dimension	 $d = 1$	 $d = 2$	 $d = 3$	d-dimensional Torus (Lattice) $(d \geq 4)$
macroscopic disorder $\frac{1}{N}\sum_{n\neq 1} 1/\lambda_n$ 1[st]-order consensus	N	$\log(N)$	bounded	bounded

- Node degree does not quantify this phenomenon
- e.g. compare  with 

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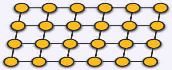
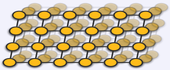
Convergence Time $1/\lambda_2$	N^2	N	$N^{2/3}$	$N^{2/d}$
dimension	 $d = 1$	 $d = 2$	 $d = 3$	d-dimensional Torus (Lattice) $(d \geq 4)$
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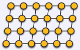
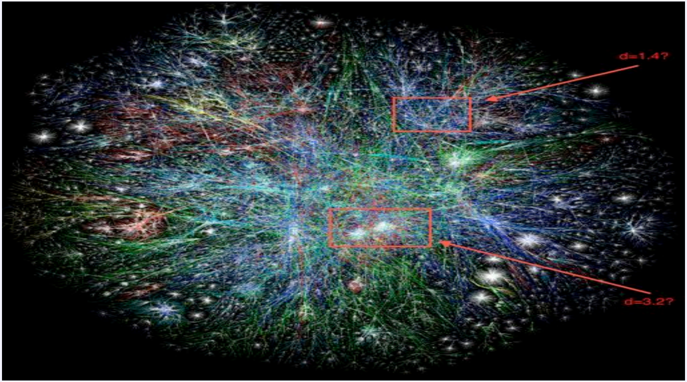
- e.g. compare  with 
- Note: $\frac{1}{N}\sum_{n\neq 1} 1/\lambda_n$ scales differently from $1/\lambda_2$

Coherence Analysis in General Graphs?

For general graphs, what is the corresponding notion of “spatial dimension”?



(opte.org)

- The *Hausdorff dimension* of a fractal graph does not fully characterize coherence

  Patterson,BB, ’11 CDC
- Open question: a *purely topological* measure of coherence for general graphs

Further Questions

- Can more general control laws break these limitations?
  - Spatially varying control gains?
  - Nonlinear feedback?
  - Dynamic feedback?
  - Asymmetric feedback?
    - ★ Improves scaling of eigenvalues as $N \to \infty$

      Barooah, Mehta, Hespanha, Hao
    - ★ but causes exponential growth (as $N \to \infty$) of system norms!!

      Tangerman, Veerman, Stosic

      Herman, Martinec, Hurak
    - ★ eigenvalues do not describe "true" system behavior

- Must have global feedback to address coherence problem
  - *Vulnerability to errors in global feedback (as $N \to \infty$)?*

Swarms and Flocks in Nature

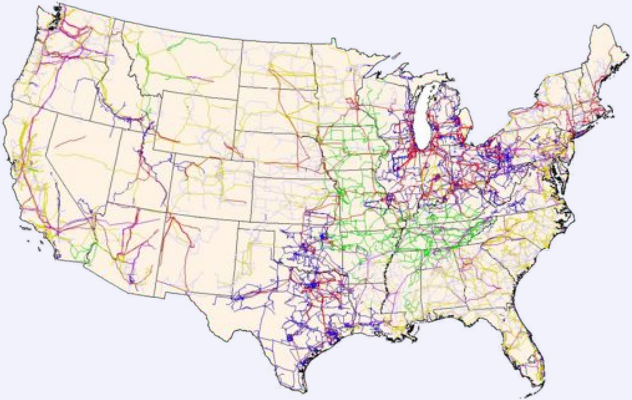
1d

2d

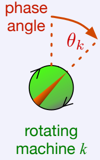
3d

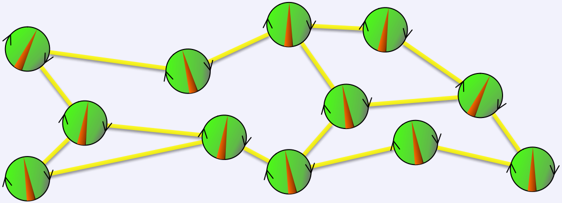
- Network dimensionality determines *coherence of motion*?
  Starling Flocks: *Young, Scardovi, Cavagna, Giardina, Leonard, '13, PLOS CB*

AC Power Networks



Phase Synchronization in AC Networks



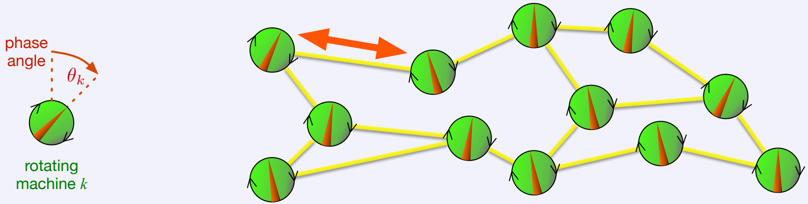


- Machines “tug” on each other to achieve phase synchrony

Linearized dynamics (swing equations) similar to vehicle formations

$$\frac{d}{dt}\begin{bmatrix}\theta\\ \omega\end{bmatrix} = \begin{bmatrix}0 & I\\ -L_B & -\beta I\end{bmatrix}\begin{bmatrix}\theta\\ \omega\end{bmatrix} + \begin{bmatrix}0\\ I\end{bmatrix} w$$

Phase Synchronization in AC Networks



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- Electrical power flows back and forth as a signaling mechanism

A Thought Experiment: Network with Identical Generators

- Assume *identical generators* but *general topology*

$$\frac{d}{dt}\begin{bmatrix}\theta\\ \omega\end{bmatrix} = \begin{bmatrix}0 & I\\ -L_B & -\beta I\end{bmatrix}\begin{bmatrix}\theta\\ \omega\end{bmatrix} + \begin{bmatrix}0\\ I\end{bmatrix} w$$
$$y = \begin{bmatrix} C_1 & 0 \end{bmatrix}$$

- Resistive power loss over (i,k) link $\quad \tilde{P}_{loss_{ik}} = g_{ik}\,|\theta_i - \theta_k|^2$
- Total resistive losses $\tilde{\mathbf{P}}_{loss} = y^* y$

$$C_1^* C_1 := L_G,$$

- Notes
  - Network Admittance Matrix: $Y = \text{Re}\{Y\} + j\text{Im}\{Y\} =: L_G + jL_B$
  - Linearized dynamics
  - Keep only quadratic part of loss term
  - Model too simple? Note: Modeling *best case scenario*, no instabilities

Calculating the H^2 Norm

Assumption: L_G is a multiple of L_B

$$\alpha := \frac{g_{ik}}{b_{ik}} = \frac{r_{ik}}{x_{ik}} = \text{ratio of line resistance to reactance}$$

Then *total resistive power loss*

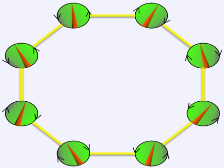
$$E\{y^* y\} = \frac{\alpha}{\beta}\,(N-1)$$

N: Network Size

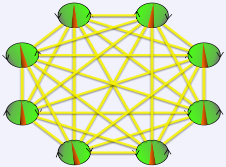
Total resistive losses are Independent of the network topology!!

Implications

Compare:

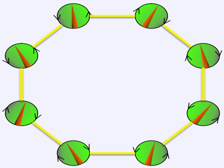
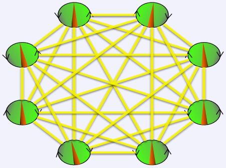


vs.



less coherent	$<$	more coherent
larger phase fluctuations	$>$	small phase fluctuations
less links	$<$	more links
Resistive losses	=	Resistive losses

Implications

Compare:  vs. 

less coherent	$<$	more coherent
larger phase fluctuations	$>$	small phase fluctuations
less links	$<$	more links
Resistive losses	=	Resistive losses

- A fundamental limitation, independent of network topology

  A consequence of using *electrical power flows* as the signaling mechanism!

  "The Price of Synchrony", BB, Gayme, '13, ACC

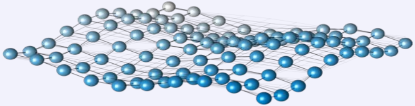
- Losses proportional to network size N

  What if $N \approx$ millions in a future highly-distributed-generation smart grid??

  Another argument for a communications layer in the smart grid

STRUCTURED, DISTRIBUTED CONTROL DESIGN

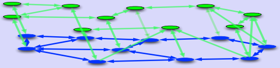
SPATIALLY DISTRIBUTED SYSTEMS

Networked/Cooperative/Distributed Control	Distributed Parameter Systems
	

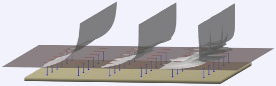
LOOK AT SPECIFIC PROBLEMS

— Vehicular Strings and Consensus

— Structured Control Design

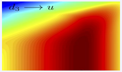


— Flow Turbulence & Control



— Spatio-temporal

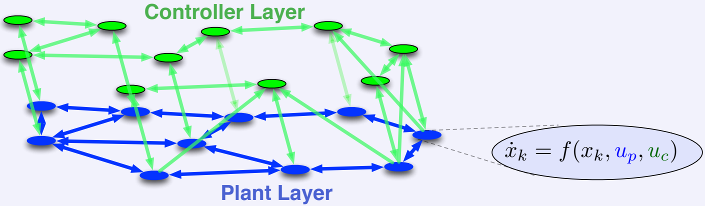
Impulse Responses



Frequency Responses

Distributed Control Systems Design

- Controller Architecture: Constraints on controller information flow



- *Optimal* Constrained Controller Design
  - In general: difficult, non-convex, non-scalable
  - Some Exceptions:
    - Partially Nested Info. Structure, Funnel Causality, Quadratic Invariance
    - Sparsity Promoting (ℓ^1-regularized) designs
  - Often possible to propose (non-optimal), scalable algorithms that "work"
    - e.g. Consensus-like algorithms (cf. multi-agent systems)

Distributed Control Systems Design

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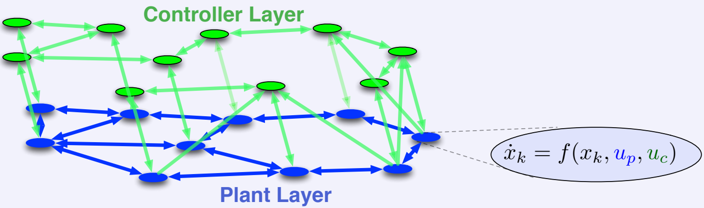

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  - Often possible to propose (non-optimal), scalable algorithms that "work"
    - ★ e.g. Consensus-like algorithms (cf. multi-agent systems)
- **Q:** Why care about optimality?

  Quantify fundamental limitations-of-performance due to *network topology*?

  akin to those due to RHP poles/zeros

Why care about difficult *optimal/robust* control problems?

- Optimality gives *Best Achievable Limits of performance*
  - e.g. a plant G with a RHP pole p and zero z

$$\inf_{C \text{ stabilizing}} \left\| (1 + PC)^{-1} \right\|_\infty = \frac{|z + p|}{|z - p|} \quad \swarrow$$

rear-steering bike:

Bicycle Dynamics and Control, K.J. Åstrom

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$$\inf_{C \text{ stabilizing}} \left\| (1+PC)^{-1} \right\|_\infty = \frac{|z+p|}{|z-p|} \quad \swarrow$$

  - If $z \neq$ p, system is both controllable/observable, the rank tests

$$\text{rank}\begin{bmatrix} \mathrm{B} & \mathrm{AB} & \cdots & \mathrm{A}^{\mathrm{n}-1}\mathrm{B} \end{bmatrix} \qquad \text{rank}\begin{bmatrix} \mathrm{C}; & \mathrm{CA}; & \cdots; & \mathrm{CA}^{\mathrm{n}-1} \end{bmatrix}$$

give a deceptive answer! (especially for large-scale systems!)

Grammians $\longrightarrow$ better measures of approximate Controllability/Observability

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$$\inf_{C \text{ stabilizing}} \left\| (1+PC)^{-1} \right\|_\infty = \frac{|z+p|}{|z-p|} \swarrow$$

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    design/characterize a good plant, not just controller design!

A point recognized in 80's-90's, but has not made it into networks literature

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  - Optimal/Robust Control is useful to
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    A point recognized in 80's-90's, but has not made it into networks literature

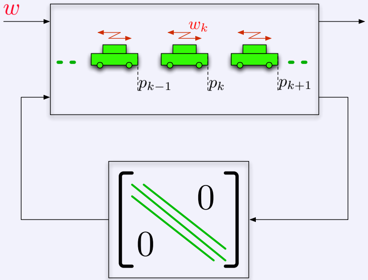
  - Use

$$\inf_{\substack{C \text{ stabilizing} \\ C \text{ structured}}} \|\mathcal{F}(G;C)\|$$

to measure *approximate network controllability/observability*

Case Study: Vehicular Formations

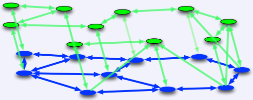
Vehicular string control with only local (no leader) information



- Corresponds to banded controller structure
- This exact problem is non-convex for any fixed N (currently unsolved)
- as $N \to \infty$
  can find lower bounds (hard performance limits) as function of topology!
- *The platoons problem is fundamentally difficult because of the 1d topology*

Structured Optimal Control in the Limit of Large System Size

- The problem $\inf_{C \text{ structured}} \|\mathcal{F}(G;C)\|$



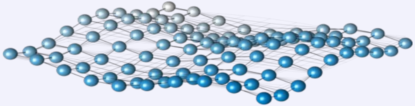
  - very difficult for finite N
  - may admit simple answers as $N \to \infty$
  - cf. *Statistical Mechanics*

- Use *structured* Robust/Optimal control problems
  not to design network controllers, but to quantify *limits of performance*

- Implications:
  - In engineered systems: allows for selection of network structures
  - In natural systems (e.g. biological): may explain naturally evolved network structures
  - *Quantify network controllability/observability*

Flow Turbulence & Control

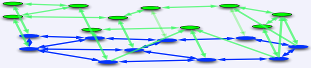
Spatially Distributed Systems

Networked/Cooperative/Distributed Control	Distributed Parameter Systems
	

Look at specific problems

— Vehicular Strings and Consensus

— Structured Control Design



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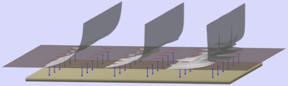

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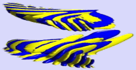

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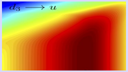
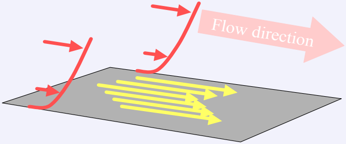
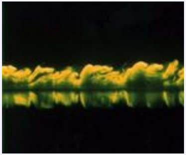


Frequency Responses

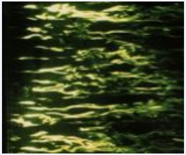
Turbulence in Streamlined Flows (Boundary Layers)



boundary layer turbulence



side view



top view

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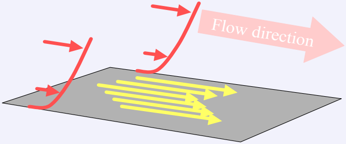
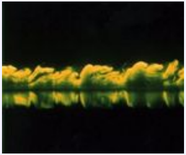


boundary layer turbulence



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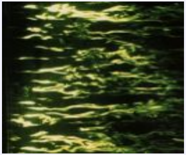

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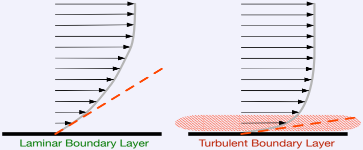


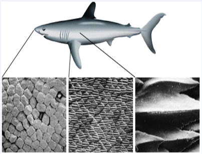
skin-friction drag: laminar vs. turbulent

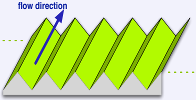
- Streamlining a vehicle reduces *form drag*
- Still stuck with: *Skin-Friction Drag* (higher in *Turbulent BL* than in *Laminar BL*)
- Same in pipe flows *(increases required pumping power)*

Control of Boundary Layer Turbulence

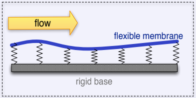
in nature: “passive” control

active control with sensor/actuator arrays

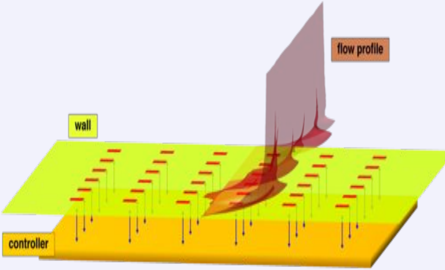




corrugated skin



compliant skin



- Intuition: must have ability to actuate at spatial scale comparable to flow structures

spatial-bandwidth of controller $\geq$ plant’s bandwidth

Control of Boundary Layer Turbulence

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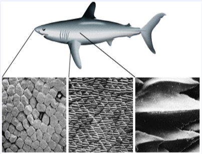

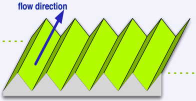

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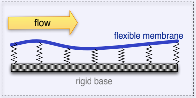

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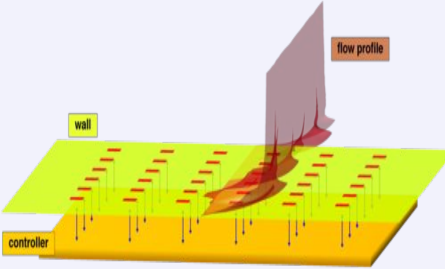


- Intuition: must have ability to actuate at spatial scale comparable to flow structures

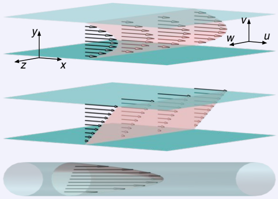
$$\text{spatial-bandwidth of controller} \quad \geq \quad \text{plant's bandwidth}$$

- Caveat: *Plant's dynamics are not well understood*

obstacles { not only device technology
also: *dynamical modeling and control design*

The Navier-Stokes (NS) equations:

$$\begin{aligned} \partial_t \mathbf{u} &= -\nabla_{\mathbf{u}}\mathbf{u} - \text{grad}\, p + \tfrac{1}{R}\Delta \mathbf{u} \\ 0 &= \text{div}\, \mathbf{u} \end{aligned}$$



- Hydrodynamic Stability: view NS as a dynamical system
- *laminar flow* $\bar{\mathbf{u}}_R :=$ a stationary solution of the NS equations (an *equilibrium*)

Mathematical Modeling of Transition: Hydrodynamic Stability

The Navier-Stokes (NS) equations:

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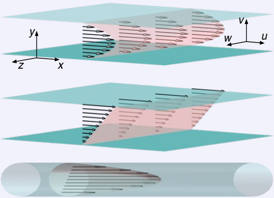


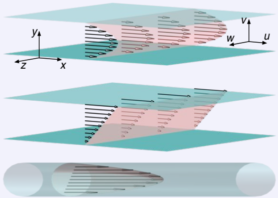
- Hydrodynamic Stability: view NS as a dynamical system
- *laminar flow* $\bar{\mathbf{u}}_R :=$ a stationary solution of the NS equations (an *equilibrium*)

laminar flow $\bar{\mathbf{u}}_R$ stable $\longleftrightarrow$ i.c. $\mathbf{u}(0) \neq \bar{\mathbf{u}}_R$, $\mathbf{u}(t) \xrightarrow{t \to \infty} \bar{\mathbf{u}}_R$

- typically done with dynamics linearized about $\bar{\mathbf{u}}_R$
- various methods to track further “non-linear behavior”

Mathematical Modeling of Transition: Hydrodynamic Stability

The Navier-Stokes (NS) equations:



$$\begin{aligned} \partial_t \mathbf{u} &= -\nabla_{\mathbf{u}} \mathbf{u} - \text{grad}\, p + \frac{1}{R} \Delta \mathbf{u} \\ 0 &= \text{div}\, \mathbf{u} \end{aligned}$$

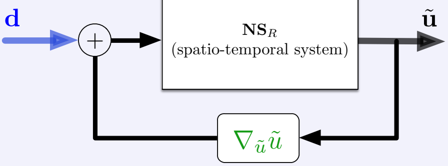
- Hydrodynamic Stability: view NS as a dynamical system
- A very successful (*phenomenologically predictive*) approach for many decades
- However: *it fails badly in the special (but important) case of streamlined flows*

- Decompose the fields as $\mathbf{u} = \underset{\text{laminar}}{\underset{\uparrow}{\bar{\mathbf{u}}_R}} + \underset{\text{fluctuations}}{\underset{\uparrow}{\tilde{\mathbf{u}}}}$

- Fluctuation dynamics: In *linear* hydrodynamic stability, $-\nabla_{\tilde{u}}\tilde{u}$ is ignored

$$\partial_t \tilde{\mathbf{u}} = -\nabla_{\bar{\mathbf{u}}_R}\tilde{\mathbf{u}} - \nabla_{\tilde{\mathbf{u}}}\bar{\mathbf{u}}_R - \text{grad}\,\tilde{p} + \frac{1}{R}\Delta\tilde{\mathbf{u}} - \nabla_{\tilde{u}}\tilde{u} + \mathbf{d}$$
$$0 = \text{div}\,\tilde{\mathbf{u}}$$

  - a time-varying *exogenous disturbance* field $\mathbf{d}$ (e.g. random body forces)



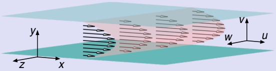
Input-Output view of the Linearized NS Equations

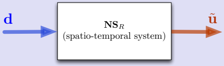
Jovanovic, BB, '05 JFM

Input-Output Analysis of the Linearized NS Equations

$$\partial_t \begin{bmatrix}\Delta\tilde{v}\\ \tilde{\omega}\end{bmatrix} = \begin{bmatrix} U''\partial_x - U\Delta\partial_x + \frac{1}{R}\Delta^2 & 0 \\ -U'\partial_z & -U\partial_x + \frac{1}{R}\Delta\end{bmatrix}\begin{bmatrix}\tilde{v}\\ \tilde{\omega}\end{bmatrix} + \begin{bmatrix} -\partial_{xy} & \partial_x^2+\partial_z^2 & -\partial_{zy} \\ \partial_z & 0 & -\partial_x\end{bmatrix}\begin{bmatrix}d_x\\ d_y\\ d_z\end{bmatrix}$$

$$\begin{bmatrix}\tilde{u}\\ \tilde{v}\\ \tilde{w}\end{bmatrix} = \left(\partial_x^2+\partial_z^2\right)^{-1}\begin{bmatrix}\partial_{xy} & -\partial_z \\ \partial_x^2+\partial_z^2 & 0 \\ \partial_{zy} & \partial_x\end{bmatrix}\begin{bmatrix}\tilde{v}\\ \tilde{\omega}\end{bmatrix}$$





$$\begin{aligned}\partial_t\Psi &= \mathcal{A}\,\Psi + \mathcal{B}\,\mathbf{d}\\ \tilde{\mathbf{u}} &= \mathcal{C}\,\Psi\end{aligned}$$

Input-Output Analysis of the Linearized NS Equations

$$
\partial_t \begin{bmatrix} \Delta\tilde{v} \\ \tilde{\omega} \end{bmatrix} = \begin{bmatrix} U''\partial_x - U\Delta\partial_x + \frac{1}{R}\Delta^2 & 0 \\ -U'\partial_z & -U\partial_x + \frac{1}{R}\Delta \end{bmatrix} \begin{bmatrix} \tilde{v} \\ \tilde{\omega} \end{bmatrix} + \begin{bmatrix} -\partial_{xy} & \partial_x^2 + \partial_z^2 & -\partial_{zy} \\ \partial_z & 0 & -\partial_x \end{bmatrix} \begin{bmatrix} d_x \\ d_y \\ d_z \end{bmatrix}
$$

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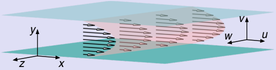

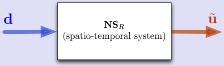

$$
\begin{aligned} \partial_t \Psi &= \mathcal{A}\,\Psi + \mathcal{B}\,\mathbf{d} \\ \tilde{\mathbf{u}} &= \mathcal{C}\,\Psi \end{aligned}
$$

- eigs $(\mathcal{A})$: determine stability

  (standard technique in *Linear Hydrodynamic Stability*)

- Transfer Function $\mathbf{d} \longrightarrow \tilde{\mathbf{u}}$: determines response to disturbances

  (uncommon in Fluid Mechanics
  an "open system")

$$\partial_t \begin{bmatrix} \Delta\tilde{v} \\ \tilde{\omega} \end{bmatrix} = \begin{bmatrix} U''\partial_x - U\Delta\partial_x + \frac{1}{R}\Delta^2 & 0 \\ -U'\partial_z & -U\partial_x + \frac{1}{R}\Delta \end{bmatrix} \begin{bmatrix} \tilde{v} \\ \tilde{\omega} \end{bmatrix} + \begin{bmatrix} -\partial_{xy} & \partial_x^2 + \partial_z^2 & -\partial_{zy} \\ \partial_z & 0 & -\partial_x \end{bmatrix} \begin{bmatrix} d_x \\ d_y \\ d_z \end{bmatrix}$$

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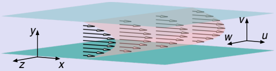
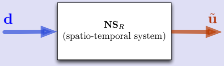




$$\begin{aligned} \partial_t \Psi &= \mathcal{A}\,\Psi + \mathcal{B}\,\mathbf{d} \\ \tilde{\mathbf{u}} &= \mathcal{C}\,\Psi \end{aligned}$$

Surprises:

- Even when $\mathcal{A}$ is stable — the gain $\mathbf{d} \longrightarrow \tilde{\mathbf{u}}$ can be very large

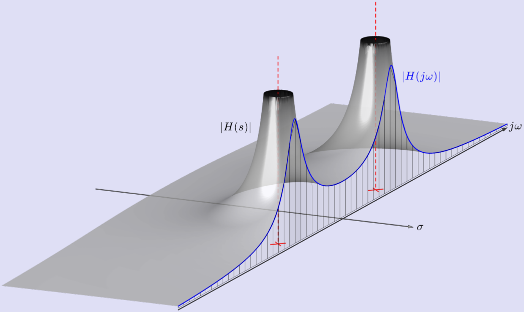
  ($(H^2$ norm$)^2$ scales with R^3)

- Input-output resonances — very different from least-damped modes of $\mathcal{A}$

Modal vs. Input-Output Response

Typically: underdamped poles $\longleftrightarrow$ frequency response peaks

cf. The “rubber sheet analogy”:



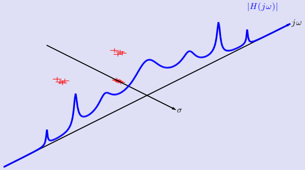
Modal vs. Input-Output Response

However: Pole Locations $\not\leftrightarrow$ Frequency Response Peaks

Theorem: *Given any desired pole locations*

$$z_1, \ \ldots, \ z_n \ \in \mathbb{C}_- \ \textit{(LHP)},$$

and any stable frequency response $H(j\omega)$, arbitrarily close approximation is achievable with



$$\left\| H(s) \ - \ \left(\sum_{i=1}^{N_1} \frac{\alpha_{1,i}}{(s-z_1)^i} \ + \ \cdots \ + \ \sum_{i=1}^{N_n} \frac{\alpha_{n,i}}{(s-z_n)^i} \right) \right\|_{\mathcal{H}^2} \leq \epsilon$$

by choosing any of the N_k's large enough

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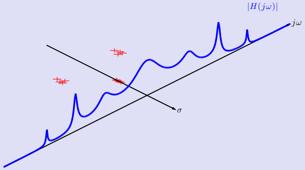


Remarks:

- No necessary relation between *pole locations* and *system resonances*
- ($\epsilon \to 0 \Rightarrow N_k \to \infty$), i.e. this is a *large-scale systems* phenomenon
- Large-scale systems: IO behavior not always predictable from modal behavior

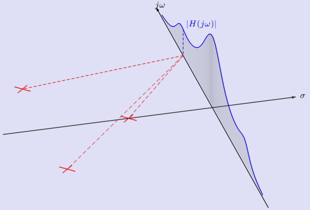
Modal vs. Input-Output Response

However: Pole Locations $\not\leftrightarrow$ Frequency Response Peaks

MIMO case: $H(s) = (sI - A)^{-1}$

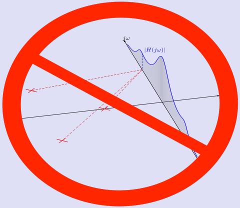
- *If A is normal* (has orthogonal eigenvectors), then

$$\sigma_{\max}\left((j\omega I - A)^{-1}\right) = \frac{1}{\text{distance}\,(j\omega,\ \text{nearest pole})}$$

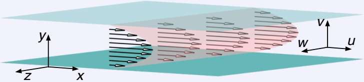


- *If A is non-normal* : no clear relation between

  singular value plot $\not\leftrightarrow$ *eigs(A)*



Translation invariance in x & z implies



- *Impulse Response* *(Green's Function)*

$$\tilde{\mathbf{u}}(t,x,y,z) = \int G(t-\tau, x-\xi, \text{y,y'}, z-\zeta)\ \mathbf{d}(\tau,\xi,y',\zeta)\ d\tau d\xi dy' d\zeta$$

$$\tilde{\mathbf{u}}(t,x,.,z) = \int \mathcal{G}(t-\tau, x-\xi, z-\zeta)\ \mathbf{d}(\tau,\xi,.,\zeta)\ d\tau d\xi d\zeta$$

$\mathcal{G}(t,x,z)$: *Operator-valued* impulse response

- *Frequency Response*

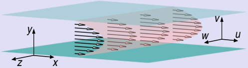
$$\tilde{\mathbf{u}}(\omega,k_x,k_z) = \mathcal{G}(\omega,k_x,k_z)\ \mathbf{d}(\omega,k_x,k_z)$$

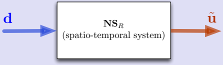
$\mathcal{G}(\omega,k_x,k_z)$: *Operator-valued* frequency response (Packs lots of information!)

- *Spectrum of* $\mathcal{A}$:

$$\sigma(\mathcal{A}) = \overline{\bigcup_{k_x,k_z} \sigma\left(\hat{\mathcal{A}}(k_x,k_z)\right)}$$

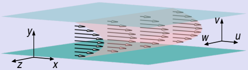
Modal vs. Input-Output Analysis

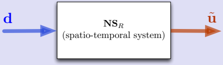




- $$\begin{aligned} \partial_t \Psi &= \mathcal{A}\,\Psi + \mathcal{B}\,\mathbf{d} \\ \tilde{\mathbf{u}} &= \mathcal{C}\,\Psi \end{aligned}$$
- IR: $\mathcal{G}(t, x, z)$
- FR: $\mathcal{G}(\omega, k_x, k_z)$

Modal vs. Input-Output Analysis





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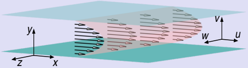
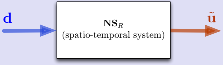
Modal Analysis: Look for unstable eigs of $\mathcal{A}$ $\left(\bigcup_{k_x,k_z} \sigma\left(\hat{\mathcal{A}}(k_x, k_z)\right)\right)$

Flow type	Classical linear theory R_c	Experimental R_c
Channel Flow	5772	$\approx$ 1,000-2,000
Plane Couette	∞	$\approx$ 350
Pipe Flow	∞	$\approx$ 2,200-100,000

Modal vs. Input-Output Analysis





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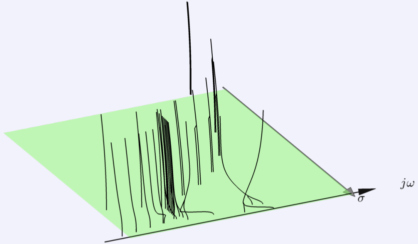
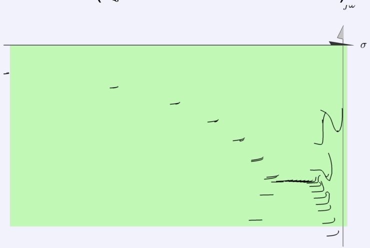
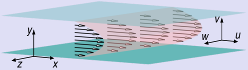
- Channel Flow @ $R = 2000$, $k_x = 1$, (k_z = vertical dimension):

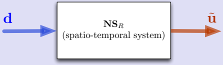




top view

Modal vs. Input-Output Analysis





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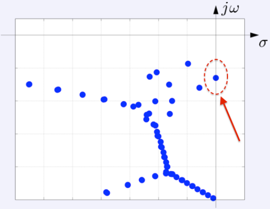
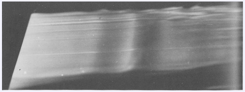
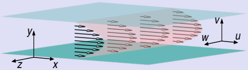
- Channel Flow @ $R = 6000$, $k_x = 1$, $k_z = 0$:

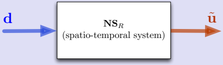


- Flow structure of corresponding eigenfunction: Tollmein-Schlichting (TS) waves



Modal vs. Input-Output Analysis





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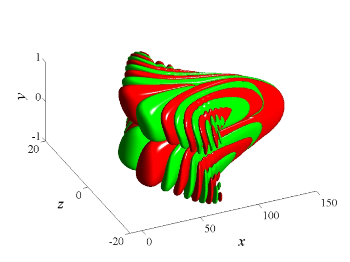
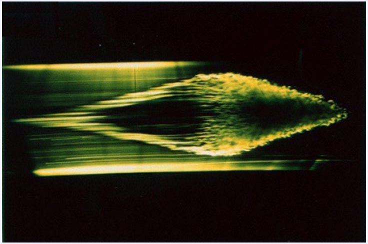
Impulse Response Analysis: Channel Flow @ $R = 2000$





similar to "turbulent spots"

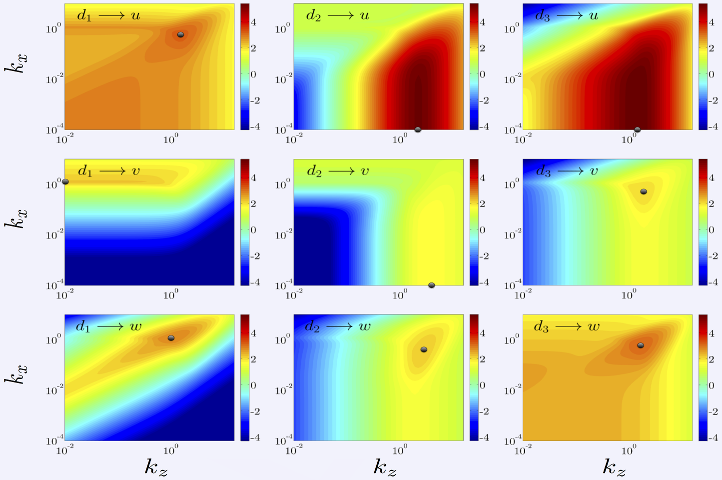
Jovanovic, BB, '01 ACC,

more movies and pics at `http://engineering.ucsb.edu/~bamieh/pics/impulse_page.html`

Spatio-temporal Frequency Response

$\mathcal{G}(\omega, k_x, k_z)$ is a *LARGE* object! (very "data rich"!)

one visualization method: $\sup_\omega \sigma_{\max}\left(\mathcal{G}(\omega, k_x, k_z)\right)$



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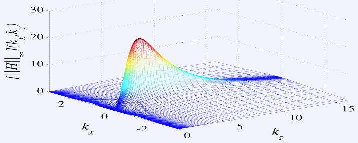
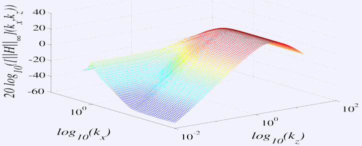
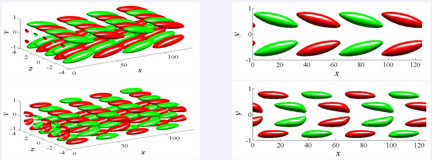
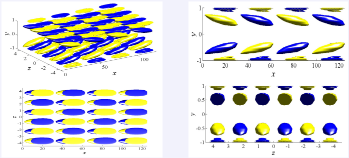




What do the corresponding flow structures look like?



streamwise velocity isosurfaces



streamwise vorticity isosurfaces

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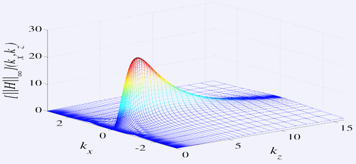
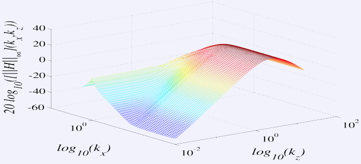
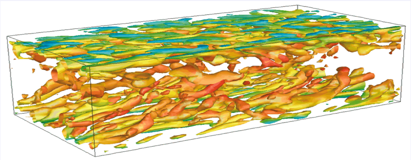
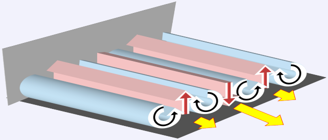




What do the corresponding flow structures look like?

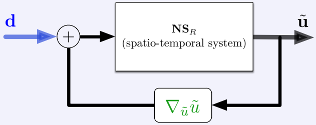
much closer (than TS waves) to structures seen in turbulent boundary layers



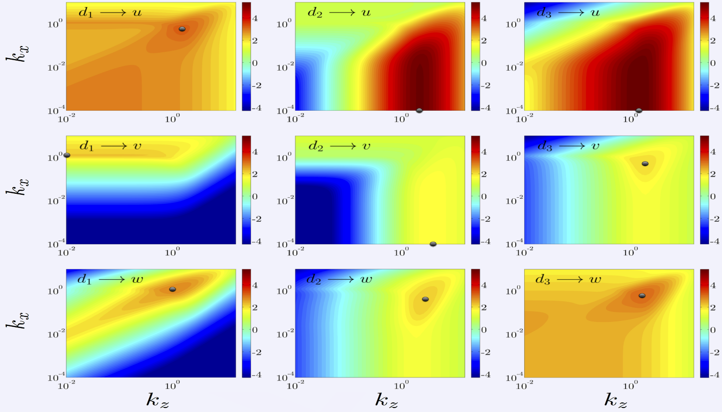


Spatio-temporal Frequency Response

How to view of $\mathcal{G}(\omega, k_x, k_z)$?



bring $\nabla_{\tilde{u}}\tilde{u}$ back in through IQCs?



Flow Control

Some recent related progress in Fluid Dynamics and Controls communities

- Farrell & Ioannou
- Henningson & Co. @ KTH
- Rowley & Co. @ Princeton
- Gayme, Doyle, Papachristodoulou & Mckeon @ Caltech
- Jovanovic & Co. @ Minnesotta

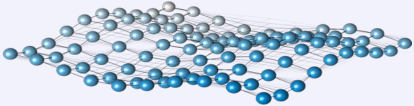
Viscoelastic turbulence
Vibrational Control with Wall Oscillations

FLOW CONTROL remains

- an *under-explored* field
- with many *high-payoff* possibilities
  - Flow and separation control
  - Control of MHD instabilities (in plasmas and liquid metals)
  - Thermoacoustics

Recap

SPATIALLY DISTRIBUTED SYSTEMS

Networked/Cooperative/Distributed Control	Distributed Parameter Systems
	

SOME COMMON THEMES EMERGE

- *The use of system norms and responses*
- *Large-scale & Regular Networks* $\longrightarrow$ *Asymptotic statements (in system size)*
- *Network topology imposes asymptotic "hard performance limits"*
- *Large-scale (even linear) systems exhibit some surprising phenomena*
- This is a very rich area with many remaining
  - fascinating questions, unsolved problems
  - research problems yet to be properly formulated

Collaborators

- M. Jovanovic
- D. Gayme
- S. Patterson
- J.C. Doyle
- B. Mckeon
- M. Dahleh
- P. Mitra
- P. Voulgaris
- F. Paganini
- M.A. Dahleh

Support:

Energy, Power & Adaptive Systems Program (ECCS)

Control Systems (CMMI)

Physics of Living Systems (PHY)

Dynamics & Control Program