The Rijke Tube

Investigating ThermoAcoustic Dynamics and Control

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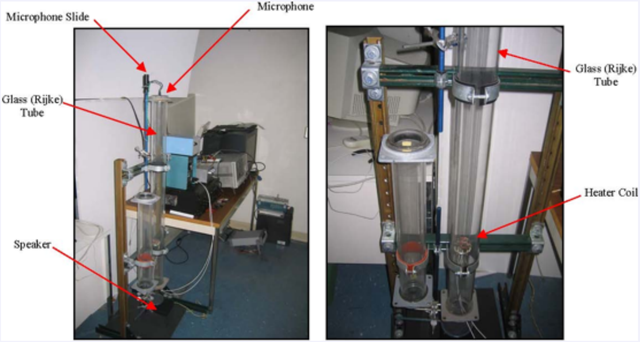
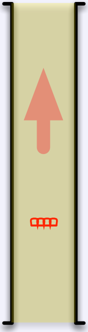
SpongFest 2012, UTD

Outline

THE RIJKE TUBE EXPERIMENT, EMPIRICALLY

The Rijke Tube Experiment

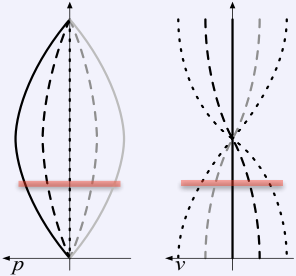


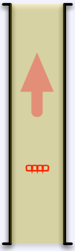


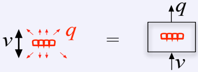
Observations:

- Heated coil induces convective flow upwards
- When coil is sufficient hot, tube begins to hum loudly

Observing the ThermoAcoustic Instability

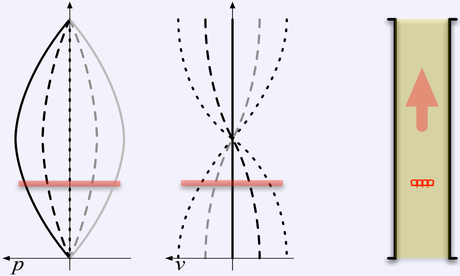


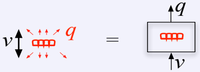




Acoustic mode: standing wave
1/2 wavelength $= L$

Observing the ThermoAcoustic Instability





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depends on velocity fluctuations v
(*convective heat transfer*)

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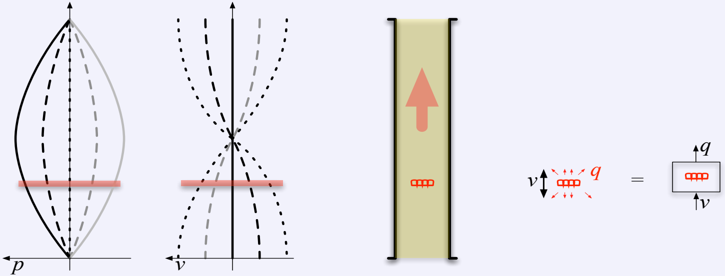

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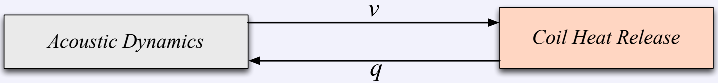
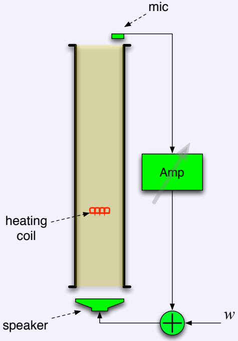
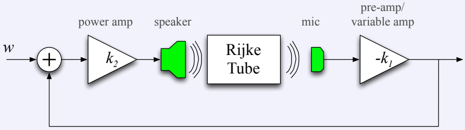


TheromAcoustic Instabilities are caused by an unstable feedback between Acoustic Dynamics and a Flow-Dependent Heat Release Process

Empirical (ID-based) Approach to the Rijke Tube





With simple *proportional acoustic feedback* K, can stabilize this system!

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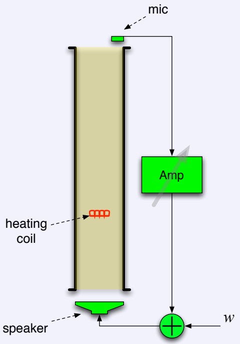

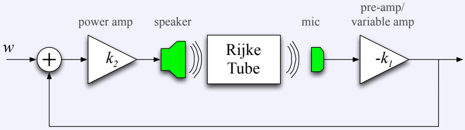

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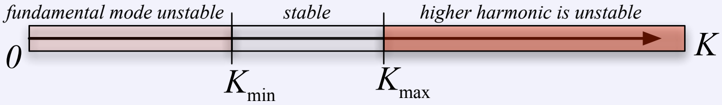
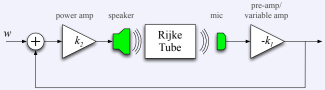
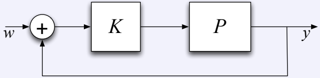


Observation: Which higher harmonic becomes unstable at high gain (e.g. 3'rd or 5'th, ..) depends on mic placement!

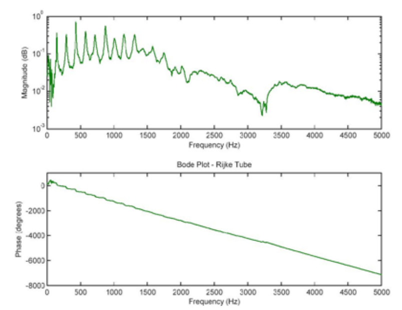
Identification of the Rijke Tube

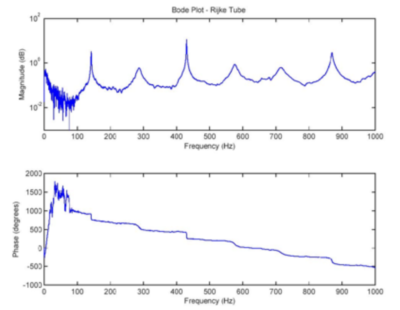




- Identify *Stabilized Closed Loop* $\quad T := \dfrac{PK}{1 - PK}, \qquad K \in [K_{\min}, K_{\max}]$
- calculate $\quad P = K \dfrac{T}{1 + T}$

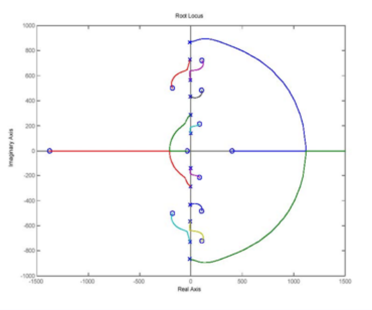
  K is uncalibrated $\rightarrow$ can determined poles and zeros of P, but not OL gain

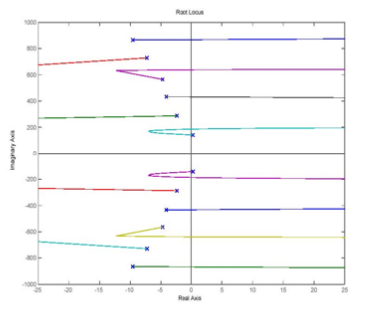




Identification of the Rijke Tube (cont.)

Validating a frequency-fitted, finite-dimensional, open-loop Rijke Tube model

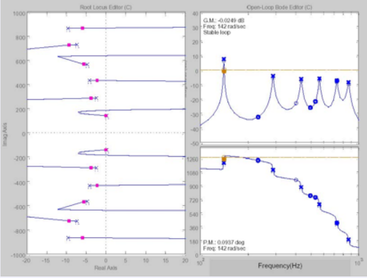


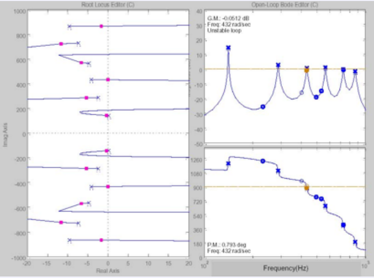


- Unstable fundamental, stable higher harmonic modes
- Parabolic root pattern characteristic of wave equation with Kelvin-Voigt damping
- OL zero locations depend on mic location, OL poles do not

Identification of the Rijke Tube (cont.)

Further validation: *which harmonic is predicted to go unstable at high gain?*





- Identified model must predict particular higher harmonic instability

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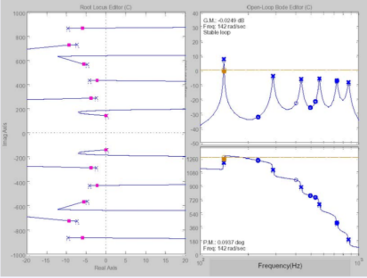
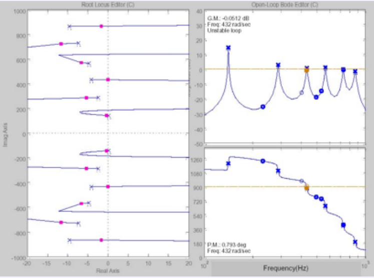
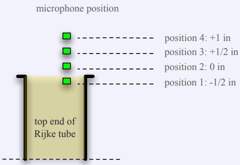




- Identified model must predict particular higher harmonic instability
- Changing mic location allows for repeat experiments



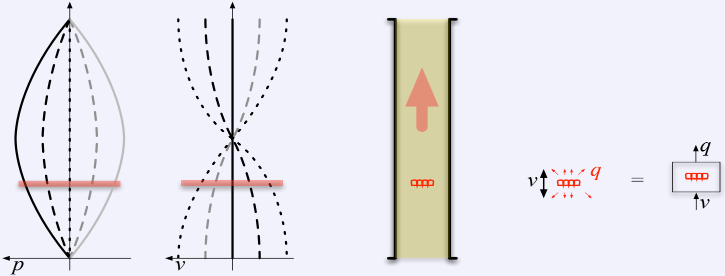
The Rijke Tube as a Controls Laboratory Experiment

- A versatile experiment to illustrate many important controls concepts
- Easy/cheap to build
- Very rich and deep (see analysis section)

cf. upcoming *Control Systems Magazine* article (with K. Åstörm)

ThermoAcoustic Energy Conversion

Basic Physics of the ThermoAcoustic Instability



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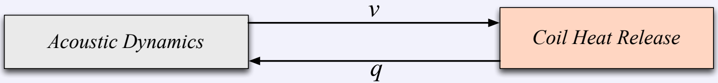

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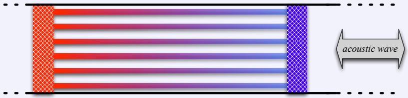
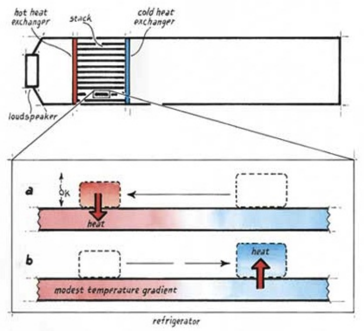
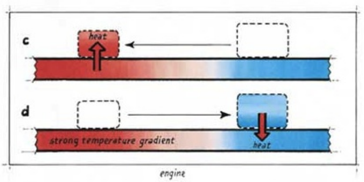
ThermoAcoustic Engines and Heat Pumps

Stack-based systems:
Temperature gradients inside acoustic cavities



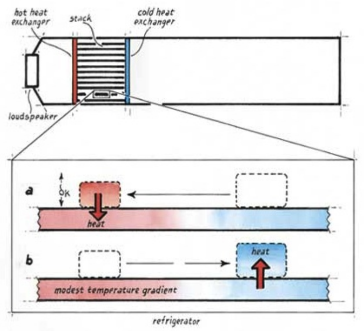
ThermoAcoustic Engines and Heat Pumps

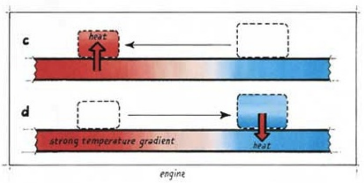




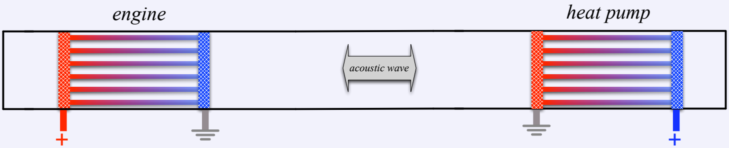
Similar principle as “pulse-tube refrigerators”

ThermoAcoustic Engines and Heat Pumps





A heat-powered refrigerator with no moving parts!



ThermoAcoustic Engines and Heat Pumps

Vision:

Replace traditional piston/crank/turbines with pressure/displacement waves

"The power of sound", Garrett & Backhaus, American Scientist, 2000

Why is this not more widely known?

Why hasn't there been more progress?

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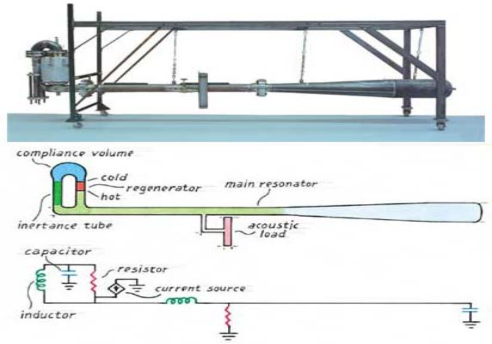
The Los Alamos Prototype:



Acoustic impedances and couplings very carefully designed!

Mechanical design is for *one* operating point!

ThermoAcoustic Engines and Heat Pumps

Vision:

Replace traditional piston/crank/turbines with pressure/displacement waves

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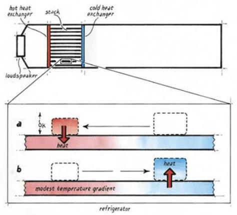
Why is this not more widely known? Why hasn't there been more progress?

- Acoustics need to adapt to changes in operating conditions
- A *delicate dance* between pressure, displacement and solid/gas heat transfer with no kinematics to enforce timing

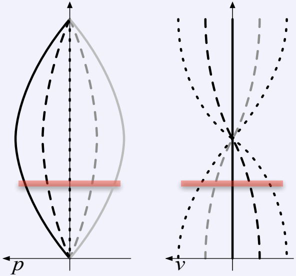


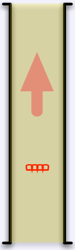
- Answer: *Active Control* is needed!
- Very little work has been done in this area

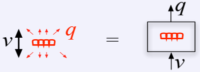
"[tunable thermoacoustic cooler]", Li, Rotea, Chiu, Mongeau, Paek, '05

ThermoAcoustic Dynamics & Control

Acoustic & Thermal Dynamics

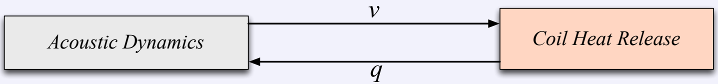




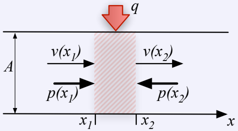


Acoustic Dynamics

Velocity-dependent heat release



1d Gas Dynamics



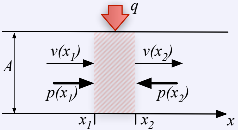
Pressure form: (there's an equivalent Temperature form)

$$\frac{\partial}{\partial t}\begin{bmatrix}\rho\\ v\\ p\end{bmatrix} = -\begin{bmatrix} v & \rho & 0\\ 0 & v & \frac{1}{\rho}\\ 0 & \gamma p & v\end{bmatrix}\begin{bmatrix}\partial_x\rho\\ \partial_x v\\ \partial_x p\end{bmatrix} + \begin{bmatrix}0\\ \beta\frac{v}{\rho}\\ \bar{\gamma}q\end{bmatrix} : \begin{bmatrix}\text{mass conservation}\\ \text{momentum balance}\\ \text{energy balance}\end{bmatrix}$$

A PDE in the Hamilton-Jacobi form with q as source term

Linearizations behave like Wave Equation

1d Gas Dynamics



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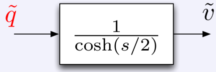
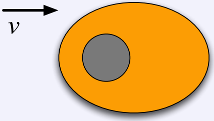


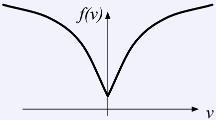
Point-to-point Transfer Function (non-dimensionalized)

Convective Heat Transfer

Hot-wire in cross flow



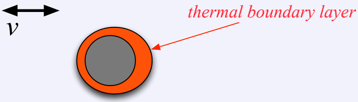
$$q_e = (T_s - T)\left(\kappa_d + \kappa_v\sqrt{|v|}\right) =: (T_s - T)f(v),$$



"King's Law" (for steady flow)

Convective Heat Transfer (unsteady)

Hot-wire in *fluctuating* cross flow



Frequency response

Lighthill '53

The response of laminar skin friction and heat transfer to fluctuations in the stream velocity

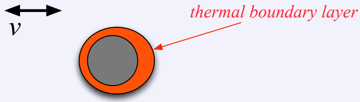
By M. J. Lighthill, F.R.S.

(*Received* 31 *December* 1953)

Convective Heat Transfer (unsteady)

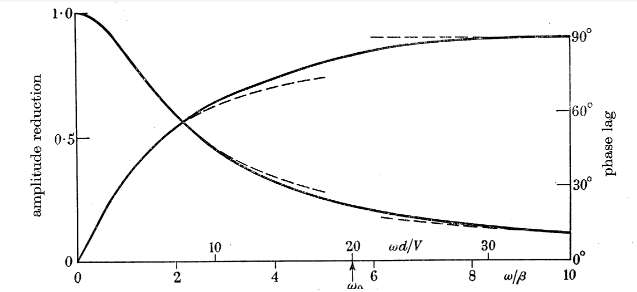
Hot-wire in *fluctuating* cross flow

Frequency response



Lighthill '53

Calculation of the frequency response (matched asymptotic expansions)



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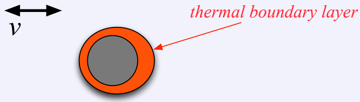


Frequency response

Lighthill '53

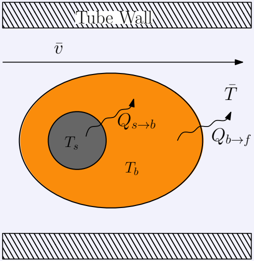
Approximate first-order lag with time delay

Special attention is paid to the phase lag in the heat transfer from a heated circular wire in a fluctuating stream, in the range of Reynolds number for which a laminar boundary layer exists. Curves for the amplitude and phase of the heat-transfer fluctuations as a function of frequency are given in figure 4, from calculations for the layer of nearly uniform thickness, which covers the front quadrant of the wire, and across which most of the fluctuating part of the heat transfer is believed to occur. For frequencies small compared with $\omega_0 = 20V/d$ (where d is the diameter), the departure of the heat-transfer fluctuations from their quasi-steady form consists essentially of a time lag of the order of $0{\cdot}2d/V$.

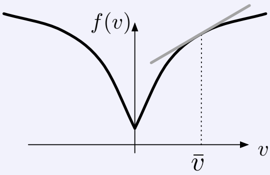
Convective Heat Transfer (a simple model)

- Use King's law for heat transfer into a *thermal boundary layer*
- Conductive heat transfer from *thermal boundary layer* to *bulk flow*

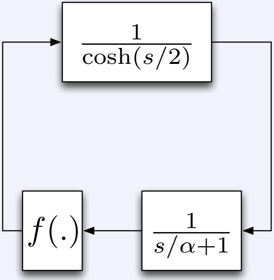
This introduces a "thermal inertia" around wire

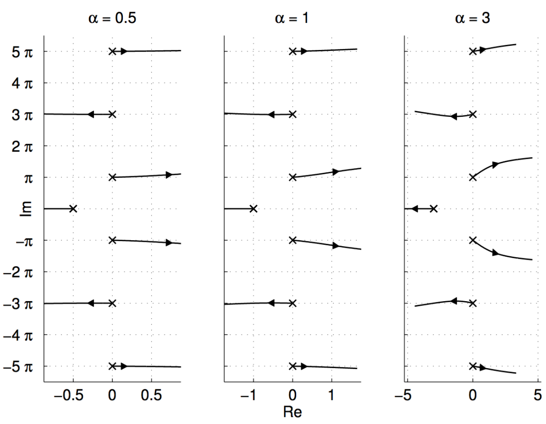


Thus:



Predicting the ThermoAcoustic Instability





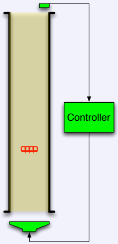
Correct prediction of the unstable fundamental mode

Optimal Periodic Control

Control to *Enhance* Heat-to-Acoustic Power Conversion

- Design control to enhance heat-to-acoustic power conversion
- Trade off control effort
- Design of an engine's "optimal limit cycle"
- Details of heat transfer are probably important
- *Similar to problems in (periodic) motion planning*

  WALKING ROBOTS $\leftrightarrow$ HUMMING ENGINES



Optimal Periodic Control

- More popular in days past ('70s)

  (*is cycling (periodic) operation better than steady (constant) operation?*)
- Approaches
  - Periodic Boundary Value Problems
  - Flatness-based

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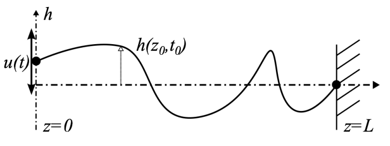
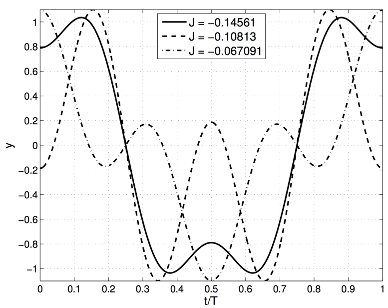
Our current approach:
Frequency domain and flatness-based method for PDE systems

Example:
non-sinusoidal operation may be optimal





("A frequency domain method for optimal periodic control", Epperline, BB, ACC '12)

Concluding Thoughts

- # of times the word “thermoacoustic” appears in the ACC or CDC proceedings (all PDF files) in the past 2 years?

  $\approx$ 2-3

- This area deserves more attention